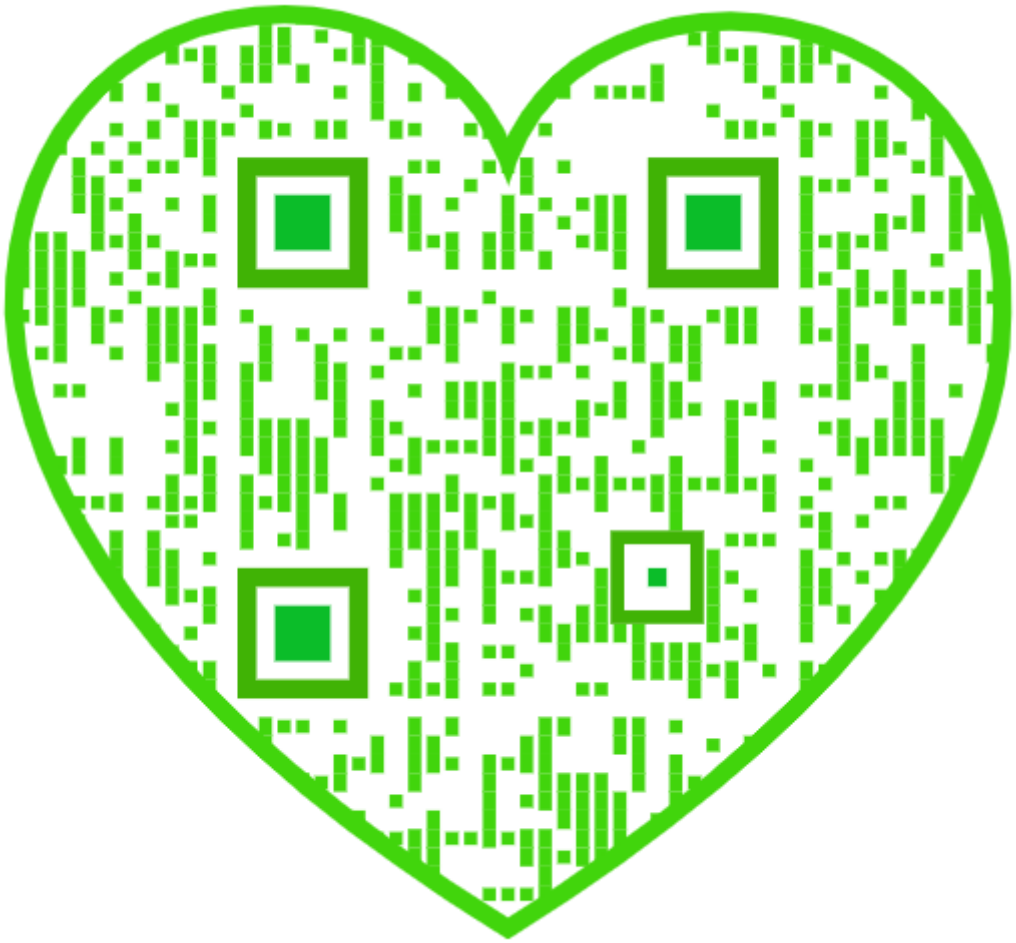


GRAHAM'S

ELEMENTARY ARITHMETIC

FOR

FOURTH AND FIFTH GRADES.

By J. W. GRAHAM.

AUTHOR OF "THIRD GRADE LESSONS IN ARITHMETIC," "SECOND GRADE LESSONS IN ARITHMETIC," "JESSIE NUMBER CARD," AND "THE PEARL SPEAKER."

[COPYRIGHTED BY J W GRAHAM.]

B. F. JOHNSON PUBLISHING COMPANY, RICHMOND, VA.
1895.

Educ T 118.95.430
HARVARD COLLEGE LIBRARY
GIFT OF
GINN AND COMPANY
DEC. 26, 1923

SHORT REVIEW OF PRIMARY RULES.

The following twelve lessons will furnish a review of the rules of the Primary Arithmetic to freshen and restore what may have been forgotten during vacation. A youth of advanced years entering school without preparation may successfully begin with this book.

I.

1. Add 3654, 8769, 5738, 376, 29,437.
2. Add 48, 976, 4389, 5050, 6573, 9875.
3. Subtract 79 from 711 until nothing remains.
4. Multiply the following numbers by 2 and add your answers: 3756, 8756, 9764, 9706, 7589, 37504, 46704.
5. Divide the following numbers by 2: 4768, 47696, 45050, 37606, 67304, 11000. Add your quotients.

(The teacher should require pupils to write name of every term in these and following examples.)

II.

1. Add 537, 69504, 48, 53073, 476955, 908703. Prove.
2. Take 683 from 6147 until nothing remains.
3. Multiply numbers in first example by 2. Add your answers.
4. Divide numbers in first example by 3. Add your quotients. Mark every term in the lesson.

III.

1. Add 35072, 87616, 34768, 75060, 378948, 707072. Prove.
2. Subtract 579 from 5211 until nothing remains.
3. Multiply the numbers in first example by 23.
4. Multiply the numbers in first example by 32.
5. Divide numbers in first example by 4. Add quotients and prove.

IV.

1. Divide the following numbers first by 2, and then by 3, and then by 4: 8769, 48956, 387504, 8765, 330375, 6575, 4084.
2. Subtract 356 from 3916 until nothing remains.
3. Multiply the following numbers—first by 23, and then by 32: 487, 9762, 4876, 4878. Find sum of all your answers.

V.

1. Multiply following numbers—first by 34, then by 43, then by 42: 6703, 47609, 87604. Find the sum of the twelve products.

2. Divide following numbers by 3, then by 4: 6704, 37695, 87695, 87695, 876074.

3. Divide numbers in second example by 5.

VI.

1. Multiply following numbers by 45, and then by 54, then by 53: 67895, 3798, 4762, 378.

2. Divide the following numbers by 5, and then by 4: 68954, 37695, 876, 9848, 650367, 860864.

VII.

1. Multiply following numbers by 56 and by 67: 87322, 476954, 76954. Add answers.

2. Divide following numbers by 6: 3742, 87695, 390872, 46895, 673284, 8956.

3. Subtract 467 from 3736 until nothing remains.

VIII.

1. Multiply following numbers by 6, then by 7, then by 8: 87324, 46939, 893246. Add answers. Mark all the terms.

2. Divide following numbers by 6, and by 7, and by 8: 36954, 87695, 37694, 476953. Mark all the terms.

IX.

1. Subtract 3759 from 30072 until nothing remains. Mark all the terms.

2. Multiply following numbers by 6, and by 7, and by 8: 36795, 3476, 87695, 37648. Add your twelve products.

3. Divide the following numbers by 6, by 7, and by 8: 30132, 8769, 37695, 47695.

X.

1. Subtract 2468 from 22212 until nothing remains. Mark all the terms. Write the difinition of addition; of subtraction; multiplication; division.

2. Multiply following numbers by 8, and by 9: 376, 476, 789, 398, 6785, 3376. Mark all the terms.

XI.

1. Multiply following numbers by 89, then by 98: 3674, 8705, 3719, 6503. Mark all the terms.

2. Define minuend, subtrahend, remainder, and mark these terms in an example of your own.

3. Define multiplicand, multiplier, product, and mark these terms in an example of your own.

XII.

1. Multiply following numbers by 8, and by 9: 376, 4835, 7695, 7656. Mark all the terms.

2. Divide answers in first example by 8, and by 9.

3. Define dividend, divisor, quotient.

(The teacher should make children thorough in the drill of these twelve lessons. There will be abundant time for completing the first half of the book, because every principal is so thoroughly and so frequently reviewed that there is hardly any need of making special review. Teachers, however, will use their own discretion.)

TO TEACHERS.

THIS book presupposes that the pupils using it have been trained in the author's "Third Grade Lessons," or have had similar training from other sources.

It is hoped and confidently urged that instructors will carry out every suggestion and direction. The little trouble entailed thereby will be repaid many fold.

FOURTH GRADE EXERCISES.

I.

1. What is division?
2. 2)4786 2)8734 2)8432 2)78346 2)93106
3. 2)391056 2)9546 2)87356 2)47858 2)53198
4. Multiply 546 by 23. Multiply 23 by 546.
5. Divide 12558 by 23.

Divisor. Dividend. Times or Quotient.

23) 12558 (

(1) See how many times.
(2) Multiply.
(3) Subtract.
(4) Bring down.

(The teacher will place two pupils at the board, with the example before them, and require *each to perform the steps as successively read by different members of the class*, according to the following procedure.)

(1) We are required to find how many times 23 is contained in 12558.

(2) *Or* 23 in 125.

(Teacher will observe to place a dot above and between the figures containing the divisor and the other figures of the dividend.)

(3) *Or* 2 in 12.

(Place the 6 in the quotient, multiply, and let the pupils see that 6 times 23 is greater than 125. Let them determine whether 5 is the figure required.)

(4) Multiply 23 by 5.

(5) Subtract 115 from 125.

(6) Bring down 5 from the dividend.

(7) We are required to find how many times 23 is contained in 105.

(8) *Or* 2 in 10.

(Pupils determine by discussion.)

(9) 4 times.

(10) Multiply 23 by 4.

(11) Subtract 92 from 105.

(12) Bring down 8 from the dividend.

(13) We are required to find how many times 23 is contained in 138.

(14) *Or* 2 in 13.

(15) 6 times.

(16) Multiply 23 by 6.

(17) Subtract 138 from 138.

(18) 23 is contained 546 times in 12558.

(19) Proof

23
546 times.
138
92
115
12558

II.

1. How do you get the half of an apple? How do you get the half of a number? Illustrate.

2. I had 100 dollars; I lost one-half of it, then spent one-half the remainder; how much had I left?

3. Divide each of the following numbers by 23: 7948; 9567; 5784; 3788; 8969.

(Work in class by the order indicated above.)

4. Add 45769, 378958, 675895, 36789, 879, 675784, 738451. Prove.

(To prove, find the sum of all the numbers; find the sum of all the numbers except the upper number; the difference between the sums will be equal to the upper number.)

III.

1. I have $1500 in bank; if I draw out $279 to-day, $387 to-morrow, and $305 the next day, how much of my money will remain in the bank?

2. Write the definition of dividend; the definition of divisor; the definition of quotient.

3. Make an example in subtraction and write the name of each term.

4. Divide the following numbers by 23: 13658; 19569; 21007; 31009; 37118. Prove.

IV.

1. I bought of a merchant 8 yards of cloth at 35 cents per yard; I paid him 16 dozen eggs at 15 cents per dozen. Did the eggs pay for the cloth?

2. Subtract 975 from 6825 until nothing remains.

3. $65\times78\times94=94\times65\times78$.

4. Divide each of the following numbers by 23: 8754; 7684; 95836; 4895; 95765; 90174.

V.

1. How do you get one-third of an orange? How do you get one-third of a number. Illustrate.

2. 3)8745 3)96745 3)87654 3)361573

3. 3)7142 3)22422 3)33533 3)88988

4. Add 47, 598, 9786, 874, 79329, 85179, 69587. Prove.

5. Divide 876542 by 23; 376940 by 23. Prove.

6. Divide 1963684 by 23; 165108 by 23.

7. Place the words dividend, divisor, and quotient over the proper terms in this lesson.

VI.

1. Add 87654, 5769, 4963, 874, 817659, 9098, 87141, 976569, 36794. Prove.

2. 3)47695 3)157895 3)76495 3)683781 Prove.

3. Multiply 387 by 34. Multiply 34 by 387.

4. Divide 13158 by 34.

(Teacher will follow order laid down in Lesson I.)

5. A man dying left $18000 to be divided among his widow, son, and daughter. The widow received one-half the money, the son one-third of the remainder; what did the daughter receive?

6. Divide 20567 by 32; 856795 by 34. Prove.

VII.

1. Divide each of the following numbers by 34: 8795; 19654; 10654; 27591. Mark all the terms.

2. How many quarts in 8754 pints?

3. How many pecks in 8752 quarts?

4. How many bushels in 8752 pecks?

5. Name all the articles that are bought and sold by these measures. What name would you give these measures? Write the table in full.

6. Multiply each of the following numbers by 10: 25; 29; 47; 58; 76; 254; 363; 474.

VIII.

1. Add 4735, 93075, 10101, 76563, 27, 57691, 67023, 87673. Prove.

2. Find the sum, the difference, the product, and quotient of the two numbers 25 and 225. Mark every term.

3. How do you get the sum of numbers? How do you get the quotient of two numbers? How do you

get the product of two numbers? How do you get the difference of two numbers?

4. Divide the following numbers by 34: 67637; 19763; 28314; 19265; 33914. Mark every term.

5. A farmer bought a barrel (44 gallons) of molasses at 68 cents a gallon, and paid for it in oats at 34 cents a bushel. How many bushels paid for the molasses? *Ans.* 88.

6. If 180 is minuend, 275 is dividend, what is the remainder?

IX.

1. If 8 pencils are worth 32 cents, what are 6 pencils worth?

(The teacher should not fail to have the cents and pencils before the class, and work objectively, and then write analysis.)

2. If subtrahend is 845, and remainder is 326, what is minuend? Let the terms be placed thus:

... Minuend.
845 Subtrahend.
326 Remainder.

Placing the sum above instead of underneath.

3. How do you get a product? Make your own example, and mark the product.

4. How do you get a difference or remainder? Make your own example, and mark the remainder.

5. How do you get a quotient? Make your own example, and mark the quotient.

6. Are the terms multiplicand and multiplier used in subtraction or division?

7. Define multiplicand. Define multiplier. Define product.

8. $67 \times 98 \times 77 = 77 \times 134 \times 49$.

X.

1. 4)47198 4)376912 4)976321 4)8769 Mark every term. Prove.

2. Add 37907, 46725, 98, 475, 6849, 68098. Prove.

3. Divide 189567 by 23; by 24; by 32; by 34.

4. What will 9 tablets cost if 6 tablets are worth 42 cents?

(Teacher should not fail to have tablets and cents before the class for objective work, and then write analysis.)

5. How do you get one-fourth of a pie? How do you get one-fourth of a number?

(Teacher should place a number of objects before the class, and illustrate one-half, one-third, etc.)

XI.

1. Divide the following numbers by 4: 15694, 8769, 6302, 87101, 70303, 67895.

2. Divide 371836 by 45; 683712÷4; 976003÷4.

3. Which is worth more—7½ barrels of flour at $6 a barrel, or 35 barrels of apples at $2 a barrel?

4. 8×25. 9×37. 7×28. 5×49. 6×39. Work these orally in class.

5. Divide 14596 by 45.

(Follow the order indicated in Lesson I., and continue its use until it shall seem not necessary.)

6. Add 96101, 375, 87693, 37194, 29, 978956, 3654, 6752. Prove.

7. Define dividend. Define multiplier. Define difference or remainder.

8. Make examples in addition, subtraction, multiplication, and division, and mark every term.

(Teacher should never accept careless work on the board or on paper. Require painstaking effort all the time.)

XII.

1. If one horse is worth $125, what are 57 horses worth?

2. If 45 books are worth 135 cents, what is one book worth?

3. If a house is worth $1875, and a store is worth $1785, which is worth more, and how much?

4. If a horse is worth $175 and a watch $5, which is worth more, and how much?

5. A watch is worth $275 and chain $15. What are both worth?

6. If 34 dollars will buy a suit of clothes, how many suits can you get for $204?

7. What will 84 sheep cost if one sheep is worth 7 dollars?

8. If 66 cents will buy a knife, how many cents will it require to buy 6 knives?

9. Divide the following numbers by 45: 87632; 76302; 180003; 10101.

XIII.

1. Find the sum of $1975, $27309, $83705, $675, $11011, $67385, $78954. Prove.

2. What is one-half of 24? One-third of 24? $\frac{1}{4}$ of 24? $\frac{1}{6}$ of 24? $\frac{1}{8}$ of 24?

(Use objects, without fail.)

3. Divide the following numbers by 48: 15601; 8759; 16789; 67342; 89104; 67323.

4. 6×25; 7×37; 8×15; 9×36; 5×28; 6×23.

(Work these examples at desk and orally in class.)

5. 27×10; 275×10; 38×100; 27×100; 30×100.

XIV.

1. Commit to memory this table:

DRY MEASURE.

2 pints make 1 quart,
8 quarts make 1 peck,
4 pecks make 1 bushel.

Write the names of the articles measured by this table.

2. How many pints in 80 quarts?
How many quarts in 84 pecks?
How many bushels in 750 pecks?

(Teacher should place these measures before the class and measure sawdust or sand. They can easily be borrowed for that purpose.)

3. If $30 will buy 6 hats, how many dollars will buy 5 hats?

(Teacher should not fail to have the hats and money before the class. Then write analysis.)

4. Write $5.01; $5.10; $6.07; $6.70; $8.08; $8.80; $9.02; $9.20; $10.01, $10.10; $12.25; $11.35; $12.65.

(Teacher should not fail to exhibit the actual money before the above numbers are written.)

5. How many cents are in 1 dime? How many dimes are in a dollar? How many dollars are in 1 eagle?

(Teacher should show the kinds of money and explain as to *mills*.)

6. Divide the following numbers by 45: 78956; 876325. Prove, and mark every term.

XV.

1. $57 \times 75 \times 55 = 11 \times 57 \times 375$.
2. $5)\underline{76953}$ $5)\underline{80607}$ $5)\underline{87102}$ $5)\underline{37019}$
3. $5)\underline{11201}$ $5)\underline{93679}$ $5)\underline{732705}$ $5)\underline{14102}$
4. Divide 876564 by 56; $169429 \div 56$.
5. Who knows what is meant by United States currency, or Federal money?

TABLE OF UNITED STATES MONEY.

10 mills make 1 cent,
10 cents make 1 dime,
10 dimes make 1 dollar,
10 dollars make 1 eagle.

Commit the table to memory.

6. How many cents in \$12? How many mills in 9 cents? How many mills in 90 cents?
7. How many dimes in \$5? How many dollars in 8 eagles? How many dollars in two hundred cents? How many dollars in 40 dimes? How many eagles in 50 dollars?

(Tell these orally, and at the same time work in chalk before class.)

XVI.

1. Multiply 749 by 56. Multiply 56 by 749.

2. Divide 41944 by 56. Prove your work, and mark every term in 1 and 2.
3. $31944 \div 56$; $219446 \div 56$; $161756 \div 56$.
4. Make an example containing multiplicand and multiplier; work it and name the answer.
5. Make an example containing minuend and subtrahend; work it and name your answer.

6. Make an example containing dividend and divisor; work it and give name to every term.

7. Two boys had together 40 marbles; one had 8 more than the other; how many marbles had each?

XVII.

1. Repeat the Table of Dry Measure.
   How many pecks in 4 bu. and 3 pks.?
2. How many quarts in 7 pks. and 3 qts.?
3. Repeat the Table of United States Money.
4. How many cents in \$5.30? \$6.25? \$9.75? \$8.25?
5. (Teacher will exercise the pupil in writing the following from dictation:) \$8.01; \$8.09; \$8.10; \$8.15; \$18.18; \$19,115; \$25.05; \$25,055. Add and prove.
6. 15×20; 27×20; 38×20; 49×20; 65×20.

(15 multiplied by 20 or twenty times 15. Also work these orally in class.)

7. A farmer sold 6 horses at \$125 each, 7 cows at \$70 each, and 70 sheep at \$8 each. He took his pay in hundred-dollar bills; how many bills did he receive? *Ans.* 18.

(In examples of this kind, and throughout the book, teachers should see that the multiplicand is *always* of denomination required in the product.)

XVIII.

1. Add 567924, 976546, 603748, 278943, 707079, 8769, 293249. Prove.
2. Divide 178984 by 56. Prove and mark every term.
3. Write names of terms in subtraction.
   Write names of terms in multiplication.
   Write names of terms in division.
   How do you get the sum?

4. Write the definition of addition; write the definition of subtraction; write the definition of multiplication; write the definition of division.

5. Divide the following numbers by 56: 87005; 187005; 287005; 387005.

XIX.

1. $87 \times 96 \times 45$ equals $45 \times 32 \times 261$.
   Mark all the multiplicands, multipliers, and products.
2. Subtract 978 from 4890 until nothing remains.
   Mark five minuends, five subtrahends, and four remainders.

3. You have been working what are called the FOUR ELEMENTARY RULES. Name them.

4. In reading numbers one figure makes an *order*, three figures make a *period*. Can you name the *orders* and *periods?* Go to the board and write and read your own numbers.

5. 87654 ÷ 56; 37624 ÷ 56; 84936 ÷ 56.

6. I lost 35 cents, then found 25; after which I had 47. How many cents had I at first?

XX.

1. How do you get one-fifth of an apple? Cut one in fifths.

2. How do you get the fifth of a number?

(Use objects, and divide them before the class.)

3. Find with objects one-half of 12; one-third of 24; $\frac{1}{4}$ of 28; $\frac{1}{5}$ of 30; $\frac{1}{6}$ of 42; $\frac{2}{3}$ of 12; $\frac{2}{3}$ of 24; $\frac{3}{4}$ of 28; $\frac{3}{5}$ of 30; $\frac{5}{6}$ of 42.

(The teacher is urged not to fail to work these exercises with objects. Players' checks furnish the best aid. Teachers should stretch a ten-foot wire or string nearly filled with spools, in

sight of class. It can be suspended on two nails, where it may be placed and removed as needed.)

4. Divide 87005 by 56; 48070÷56.

5. The sum of four numbers is 480. The first number is $\frac{1}{8}$ of 480; the second is one-fourth of first number; the third number is three times the second. What is the fourth number? Ans. 160.

XXI.

1. Add 57695, 710956, 8367, 971056, 83245, 679356, 8745, 4765. Prove.

2. Find the sum of 63057, 17875, 3759, 930769, 283059, 120120, 97576. Prove.

3. 976504÷56; 876954÷56.

4. Three boys, each having 25 marbles, commenced to play. After playing some time, one boy had won 6 marbles, another boy had won 5; how many did each have when they quit playing?

(Play it out in class with marbles.)

XXII.

1. Multiply 987 67
67 987

Mark multiplicands, multipliers, and products.

2. Divide 66129 by 67.

3. Write the lines beginning "Thirty days hath September," etc.

4. Divide 489125 by 67.

Prove, and mark all the terms.

5. Write the names of the Four Elementary Rules.

6. Write a number of three orders; a number of four orders; a number of five orders; a number of

two orders; a number of six orders; a number of seven orders.

7. Name the orders in the following numbers: 875; 1875; 22375; 78; 675875. Add and prove.

8. $87 \times 92 \times 65 = 65 \times 23 \times 348$.

XXIII.

1. Write out the following numbers after this model:

$$5786 \left\{ \begin{array}{r} 5000 \\ 700 \\ 80 \\ 6 \end{array} \right.$$

1625; 2342; 3794; 972; 19247.

2. When you divide a number by 4, what do you get? When you divide by 5, what do you get? When you divide by 6, what do you get? *Ans.* One-sixth.

3. Find sum, difference, product, and quotient of the numbers 37 and 222, and give every term its name.

4. Write Table of Dry Measure.

5. Write Table of Federal Money.

6. Write from dictation· $8.75; $28.50; $28.05; $3,875; $20.10; $15,206; $375.20. Add these numbers. What does "dictation" mean?

XXIV.

1. Divide the following numbers by 67: 87654; 27304; 97634; 193754. Prove each example, and mark all the terms.

2. Change 500 pints to quarts. Change 500 quarts to pints. Change 420 pecks to bushels. Change 420 pecks to quarts. Change 100 bushels to pecks.

3. Change bu. pks. qts. 5 3 4 to quarts.

bu.	pks.	qts.
5	3	4
	4	
	23	
		8
	188	

Remember this model, and always mark every term.

(Bring the measures before your class; by handling the measures pupils realize the value of the units. Do not neglect this because of the trouble. The teacher should have these and similar examples explained by strict analysis. If 1 bushel equals 4 pecks, 5 bushels equal 5 times 4 pecks, etc. Then explain that the plan of the model is for convenience, or for saving figures and shortening work.)

4. Change bu. pks. qts. pt. 25 2 5 1 to pints.

5. How long will it take two boys to count 1000 pennies, if each boy counts two pennies a second?

XXV.

1. Change the following numbers to cents: \$7.17; \$5.37; \$6.25; \$37.15; \$28.40.

2. Change 400 cents to dollars. Change \$400 to dimes. Change 540 dimes to dollars.

3. 6×37; 7×49; 8×27; 9×22; 5×48; 18×30; 15×30; 28×30; 27×30; 45×30.

Work these orally in class.

4. Change bu. pk. qts. pt. 26 1 3 1 to pints.

5. $\frac{1}{3}$ of 36; $\frac{1}{4}$ of 36; $\frac{1}{9}$ of 36; $\frac{1}{6}$ of 36; $\frac{2}{3}$ of 36; $\frac{3}{4}$ of 36; $\frac{5}{9}$ of 36; $\frac{5}{6}$ of 36.

(Teacher will work with objects.)

6. How do you get one-third of a number? one-fourth of a number? one-ninth of a number?

7. If 8 books cost 72 cents, how many books can you get for 81 cents?

(Teacher must not fail to have the books and money before the class, and then write the analysis.)

XXVI.

1. Divide the following numbers by 67: 83754; 92174; 473265; 38101; 765493; 808088.

2. Add 769543, 256976, 430065, 4769, 4673295, 853004, 76954. Prove.

3. Add the following numbers: \$8.47, \$7.65, \$5.375, \$25.65, \$36.875, \$16.625. Prove.

4. (Having had the work of the lesson handed in on paper, spend the time of the lesson in writing numbers in Federal money.)

XXVII.

1. Divide the following numbers by 67: 27354; 28351; 76950; 185605; 30697. Prove.

2. Divide 18955895, and each quotient, by 7 until you have 7895 for a quotient.

3. Change 17 bu. 3 pks. 1 qt. 1 pt. to pints.

4. If 63 cents will buy 9 pencils, how many pencils will 28 cents buy?

(Teacher must not fail to have the pencils and cents before the class.)

5. Write the Table of Dry Measure. Write what articles are measured by it.

6. $\frac{1}{2}$ of 30; $\frac{1}{3}$ of 30; $\frac{2}{3}$ of 30; $\frac{1}{5}$ of 30; $\frac{3}{5}$ of 30; $\frac{1}{6}$ of 30; $\frac{5}{6}$ of 30; $\frac{1}{10}$ of 30; $\frac{7}{10}$ of 30.

(Teacher must not fail to work these in objects before the

class. Begin to test your pupils by asking some one to tell you direct: $\frac{1}{4}$ of 24 = ?)

7. Write Table of Federal Money.

XXVIII.

1. Write the following numbers thus:

$$14529 = \begin{cases} 10000 \\ 4000 \\ 500 \\ 20 \\ 9 \end{cases}$$

25364; 37569; 38691.

2. Divide 87329 by 78. Prove, and mark every term.

3. What is the answer called in addition? In division? In multiplication? In subtraction.

4. Make an example in subtraction, and mark all the terms. Make an example in multiplication, and mark all the terms. Make an example in division, and mark all the terms.

5. Write from dictation \$87.65; \$170,055; \$395,012; \$478.10; \$384,505; \$101,101. Find the sum of these numbers, and prove.

6. Find the sum, difference, product, and quotient of the numbers 37 and 180, and mark all the terms.

XXIX.

1. Divide the following numbers by 78: 17834; 23009; 15007; 31008; 20907. Prove, and mark all the terms.

2. Change 300 pints to quarts. Change 300 pecks to quarts. Change 300 bushels to pecks. Which is a higher denomination, a pint or a quart? A peck or a bushel? A bushel or a quart?

3. How many pints in 1 peck? How many quarts

in 1 bushel? How many bushels in 64 quarts? How many bushels in 64 pints?

(Teacher will measure sawdust with these measures, before the class.)

4. Which is greater, 19 pints or 9 quarts? Which is greater, 10 pecks or 3 bushels? Which is greater, 25 pints or 2 pecks?

5. Place on the table two objects, and tell what part one object is. Place three objects, and tell what part one object is; what part two objects are; and so on with 6 objects, 8 objects, 10 objects, etc.

(Teacher should not fail to carry this out literally.)

XXX.

1. If one horse is worth $75, how many horses can you get for $375?

2. If a house is worth $375, and a horse is worth $75, how much is one worth more than the other?

3. If 375 cents will buy 75 pencils, what is one pencil worth?

4. If one cow is worth $25, what are 375 cows worth?

5. If you earn $375 and spend $75 twice, how much will you have left?

6. A boy paid 24 cents for oranges at 3 cents each; he ate 4 of them, and sold the rest at 6 cents each, did those he ate cost him any thing at last?

7. Multiply the following numbers by 10: 78; 96; 965; 37; 375; 42; 424; 385; 38; 57; 56; 72. Multiply the same numbers by 100.

(Do this also orally in class.)

XXXI.

1. Divide the following numbers by 78: 6742; 96354; 16874; 25365.

2. Divide 18753 by 23; then by 34; then by 45; then by 56; then by 67; then by 78.

3. $\frac{2}{3}$ of 18=? $\frac{3}{4}$ of 20=? $\frac{2}{3}$ of 15=? $\frac{2}{3}$ of 9=? $\frac{1}{5}$ of 10=? $\frac{2}{5}$ of 10=? $\frac{3}{4}$ of 12=? $\frac{1}{8}$ of 16=? $\frac{3}{8}$ of 16=?

(The work to be mental; result to be placed after the sign of equality.)

4. The subtrahend is 751; the remainder is 275. What is the minuend? Place the numbers thus:

... Minuend.
751 Subtrahend.
275 Remainder.

Add, and place result over instead of underneath.

5. How many trees in an orchard, if there are 25 rows, with 12 trees in each row?

(Use broom-straws for trees, and stick them in sand in a shallow box.)

6. If 180 trees are in an orchard, and 15 trees in each row, how many rows are there?

(Proceed as above.)

XXXII.

1. 8)57796 8)64325 8)101012 8)34705 8)6547

2. What are the Four Elementary Rules?

3. Make a number on the board of 6 figures, and name each order.

4. Write the Table of Dry Measure. Write Table of Federal Money.

5. How many cents in $4.50? in $5.75? in $3.75? in $12.75? in $14.25? in $18.40? Add and prove.

6. Change 200 dimes to cents; 200 cents to dollars; 200 dollars to dimes; 200 dollars to cents.

7. Write from dictation $20.205, $35.70, $29.09, $8.01, $19.015, $27.275, $27.203. Add these numbers.

XXXIII.

1. Divide the following numbers by 8: 6744; 90905; 37049; 67325; 63245; 8751.

2. $375 \times 89 = ?$ $89 \times 375 = ?$

3. Divide 33375 by 89. Prove, and mark every term. Divide 67067 by 89. Prove, and mark terms.

4. Change 475 bu. 3 pks. to pks. Change 475 bu. 3 pks. 5 qts. 1 pt. to pints. Change 475 bu. 3 qts. 1 pt. to pints.

5. What does 2 A.M. mean? What does 4 P.M. mean? How many hours from 2 A.M. to 4 P.M.

6. How many inches in 1 foot? How many feet in 1 yard? How many inches in 2 yards?

(Teacher should not fail to place foot rule and yard measure before the class. Let them make themselves acquainted with the inch, the foot, and the yard by actual use in practice.)

XXXIV.

1. Divide following numbers by 89: 7642; 9712; 25675; 37018; 49018.

2. Add 375012, 60705, 89325, 976950, 47325, 80970, 376956. Prove.

3. If 10 oranges cost 50 cents, how many oranges can you buy for 90 cents?

(The teacher should not fail to have the oranges and cents, and work before the class; then write analysis.)

4. If 10 oranges cost 50 cents, what will 25 oranges cost?

5. I lost 75 cents, found 48 cents, and then had one dollar. How much had I at first?

6. How many pounds of sugar at 14 cents a pound will pay for 35 pounds of rice at 8 cents a pound?

XXXV.

1. Divide the following numbers by 89: 8909; 8989; 90089; 39089; 59089. Mark all the terms.

2. What is the answer called in addition? in subtraction? in multiplication? in division?

3. Find the sum, product, difference, and quotient of 89 and 801. Mark all the terms.

4. Write the definition of addition; of subtraction; of multiplication; of division.

5. Write the names of the terms you use to get difference or remainder. Write the names of the terms to get a product. Write names of terms to get a quotient.

6. Change 25 bu. 1 pk. 1 qt. to pints. Mark every term as in model in Lesson XXIV., example 3.

XXXVI.

1. Place 2 objects on the table, and ask what part of 2 objects 1 object is; place 3 objects, and ask what part 1 object is; what part 2 objects are of 3. Place 4, 5, 6, 10, 15 objects, and practice in the same way.

2. Find $\frac{1}{3}$ of 18; $\frac{5}{8}$ of 18; $\frac{1}{7}$ of 14; $\frac{5}{7}$ of 14; $\frac{1}{7}$ of 28; $\frac{3}{7}$ of 28; $\frac{1}{9}$ of 36; $\frac{5}{9}$ of 36; $\frac{1}{8}$ of 32; $\frac{7}{8}$ of 32.

3. How do you get the half of any thing? How do you get the third? The fourth? The fifth? The sixth? The seventh? The eighth?

(The teacher should not fail to provide a large number of pasteboard disks of uniform size—say 5 inches in diameter—and cut them before the class. Separate objects in the same way.)

4. How do you get $\frac{1}{2}$ of a number? $\frac{1}{3}$ of a number? $\frac{1}{2}$ of 6? $\frac{1}{3}$ of 6? $\frac{1}{4}$ of a number? one-fifth of a number? one-sixth of a number? one-seventh of a number?

XXXVII.

1. Divide 19375 by 23; by 34; by 45; by 56; by 67; by 78, by 89.

2. Find difference between $80.80 and $15.75. Subtract $35.09 from $295.10. $755—$98.15. Subtract $25.025 from $30. Take 75 cents from $0.875. $37.125—$16.09. $20—$16.055.

3. Take $30.625 from eighty dollars. Take $201.01 from $375.005. $87.45 less $86.455.

4. Multiply these numbers by 89: $47.05; $27.06; $47.01; $35,076; $67.05; $65.50.

XXXVIII.

1. Divide the following numbers by 9: 60756; 715392; 84649; 93654.

2. Divide these numbers by 98: $441.98; $550.76; $614.46; $828.10.

3. Take 75 cents from one dollar and a half. Subtract $1.625 from $2; $97.35 less $96.46.

4. Multiply $26.45 by 98; $37.05 by 89.

5. A farmer sold 45 turkeys at the rate of 5 turkeys for $3. What did he get for them? *Ans.* $27.

6. A boy sold 100 oranges at the rate of 10 oranges for 25 cents; what did he receive?

7. How many men can do a work in 3 days that takes 1 man 9 days?

8. Add 47695, 30976, 763295, 909009, 60175, 478045, 1275769. Prove.

XXXIX.

1. Divide 147695 by 234. Use the order outlined in Lesson I.

2. How many quarts in 400 pints?

Mark all the terms after this model: $\overset{\text{pts.}}{2})\overset{\text{pts.}}{400}$

$\underline{200}$ qts.

3. How many bushels in 500 pecks?

How many pecks in 500 quarts?

Mark all the terms as in the model in the preceding example.

4. Reduce 41 bu. 2 pks. 1 pt. to pints.

5. Write from dictation the following numbers: \$5.01; \$25.01; \$21.05; \$31.05; \$31,011; \$57.09; \$40.40; \$5.005. Add these numbers and prove.

6. Name the orders in each of the following numbers: 870750; 1057654; 2305405. Multiply each number by 234.

XL.

1. \$80.01 less \$59.009; \$775.25—\$395.75; take \$95,-017 from \$100; take \$5.875 from \$10.

2. Divide 2107595 by 234.

Prove and mark all the terms.

3. The quotient is 753, divisor 35; what is the dividend? Arrange the work in this manner:

Proof.

Divisor.	Dividend.	Quotient.		Divisor.	Dividend.	Quotient.
35)	26355	(753		35)		(753
	245					35
	185					3765
	175					2259
	105					26355
	105					

4. I gave 48 bushels of oats for 17 gallons of molasses at 96 cents per gallon. What did I get per bushel? *Ans.* 34 cents.

5. If 15 bushels of wheat make 3 barrels of flour, how many bushels will make 7 barrels?

6. Make your own example with a multiplicand and multiplier, and mark all the terms.

XLI.

1. Make an example in subtraction, and mark all the terms.

2. A man bought 39 cows at $75.25 each and 45 horses at $90 each; how much money did he pay for all? *Ans.* $6984.75.

3. I sold 48 sheep at $3.75 each, and took in payment 48 cords of wood at $2.50 a cord and a horse. How much did the horse cost me? *Ans.* $60.

4. I spent during the year $875.25, and had $329.50 left; how much did I have at the beginning of the year?

5. A printer earned $75 per month, and spent $45 per month; how much did he save by the end of the year? *Ans.* $360.

6. Divide 275695 by 234; divide the same number by 345.

XLII.

1. $\frac{1}{2}$ of 24=? $\frac{1}{3}$ of 21; $\frac{2}{3}$ of 21; $\frac{1}{3}$ of 15; $\frac{2}{3}$ of 15; $\frac{1}{3}$ of 24; $\frac{2}{3}$ of 24; $\frac{5}{8}$ of 24.

2. Divide 725406 by 345. Mark the terms. Prove.

3. Write from dictation: $87.75; $375.09; $49.90; $484.15; $397.095; $607.375; $40,425. Add and prove.

4. What do you mean by Federal currency? Write the Table of Federal Money.

5. Write the Table of Dry Measure.

6. Reduce 5 bu., 3 pks. to pints.

7. What is a simple number? Make five simple

numbers. What is a compound number? Do you see any compound numbers in this lesson? Make five compound numbers.

XLIII.

1. Divide the following numbers by 456: 87952; 753750; 81089; 281090; 374000.

Prove each example, and mark the terms.

2. January 1, 1880.

MR. BROWN,

Bought of MR. JONES.

5 yds. of broadcloth	@	$5.75	per yard,	.....
15 yds. of domestic	@	.15	per yard,	.....
25 yds. of ribbon	@	.375	per yard,	.....
30 yds. of flannel	@	.55	per yard,	.....
			Ans.	$56.875

3. Quotient is 287, divisor is 345; what is the dividend? Prove.

(See Lesson XL., Ex. 3.)

4. Reduce 200 cents to mills; 300 mills to cents; 300 cents to dimes; 300 cents to dollars; 550 dollars to cents; 550 dollars to eagles.

XLIV.

1. Divide 375098 by 455; by 567. Prove. Mark all the terms.

2. What is a *receipt?* Write a receipt for the payment of ten dollars.

3. What is a simple number? Write five simple numbers.

4. What is a compound number? Write five compound numbers.

5. Which is greater in value, 4 pecks or 27 quarts?

27 quarts or 1 bushel? 35 pints or 3 pecks? 2 bushels or 20 pecks?

6. May 1, 1890.

MR. WILLIAM JOHNSON,

Bought of THOMAS MILLER.

57 lbs. of flour @ $ .075 per pound, ...
38 lbs. of sugar @ .125 per pound, ...
96 lbs. of nails @ .06 per pound, ...
84 lbs. of soap @ .125 per pound, ...

Ans. $\overline{\$25.285}$

7. What is *reduction?* Change 575 eagles to dollars.

XLV.

1. Divide 570932 by 567; by 678; by 707; by 808; by 909; by 505.

2. What is a coin? Write the names of the coins used in trade.

3. What are denominate numbers? Would you go to a higher or lower denomination in reducing pecks to quarts? in reducing pints to pecks? in reducing bushels to pecks? in reducing bushels to quarts? in reducing pecks to pints?

4. Show what 1 bushel equals. Show what 1 peck equals. What 1 quart equals. Arrange the above after this plan:

1 bushel=4 pecks=32 quarts=64 pints.
1 peck=8 quarts=16 pints.
1 quart= 2 pints.

5. Find $\frac{1}{4}$ of 40; $\frac{1}{5}$ of 40; $\frac{1}{8}$ of 40; $\frac{1}{10}$ of 40; $\frac{3}{4}$ of 40; $\frac{4}{5}$ of 40; $\frac{3}{8}$ of 40; $\frac{7}{10}$ of 40.

6. What does 4 A.M. mean? 4 P.M.? How long is it from 4 A.M. to 4 P.M.?

7. Would you go to a higher or lower denomina-

tion in reducing cents to dollars? in reducing mills to cents? in reducing dollars to eagles?

XLVI.

1. Divide the following numbers by 6: 87654; 378954; 248956; 756904.

2. Divide the following numbers by 67: 875412; 30757; 875934. Prove, and mark all the terms.

3. Find the sum, product, difference, and quotient of the two numbers 576 and 1728, and mark the terms in each example.

4. Repeat the

TABLE OF LONG MEASURE.

12	inches	make 1 foot (ft.).
3	feet	make 1 yard (yd.).
5½	yards	make 1 rod (rd.).
40	rods	make 1 furlong (fur.).
8	furlongs	make 1 mile (mi.).

5. Write from dictation \$50.75; \$818.091; \$76.003; \$915.017; \$937.055; \$667.101; \$87.095; \$8154. Add these numbers, and prove.

XLVII.

1. Repeat and write from memory the Table of Long Measure. For what is it used?

2. Repeat the

TABLE OF LIQUID MEASURE.

4 gills	make 1 pint (pt.).
2 pints	make 1 quart (qt.).
4 quarts	make 1 gallon (gal.).

For what is it used? There is no regular number of gallons in a barrel. A hogshead is said to hold 63 gallons.

3. Reduce 3 miles, 7 furlongs to furlongs. Reduce 40 yards, 2 feet, 10 inches to inches. Reduce 50

inches to feet; 200 feet to yards; 500 furlongs to miles; 80 rods to furlongs.

(The teacher should not fail to have the foot rule and yard measure before the class, and require them frequently to lay off the inch, foot, yard, and rod. This is the only way to impress the minds of the children with the relative values of these units of measure.)

4. Change 500 cents to dollars; 870 dimes to dollars; 37 dollars to dimes; 37 eagles to dimes; 400 eagles to cents.

5. Divide the following numbers by 7: 87694; 871024; 36756. Divide the same numbers by 78. Divide the same numbers by 789.

XLVIII.

1. If 7 men can mow a field in 5 days, how long will it take 5 men?

(Represent a day's work with a small square of paper. 7 men's work for 1 day would be represented by 7 squares; for 5 days 35 squares would be required, etc.)

2. I bought 87 bushels of wheat at 75 cents a bushel, 96 bushels of oats at 33 cents per bushel, 47 bushels of rye at 64 cents. I paid for all in cloth at 84 cents per yard, and balance in money. How many yards did I give, and how much money?

Ans. 151 yards, and 17 cents in money.

3. If a box holds 35 pounds, how many boxes will hold 24325 pounds? Prove.

4. If 1 pound of butter is worth 35 cents, what is the value of 24325 pounds?

5. If a boy read 14 pages a day, in how many days can he read 12 books of 84 pages each.

Ans. 72 days.

6. How much will 5 boys earn in 1 year, if they each earn 10 dollars a month? *Ans.* $600.

XLIX.

1. Write from memory the Table of Dry Measure. Write the Table of Liquid Measure. Write the Table of Long Measure. Write the Table of United States Money. Write what each table is used for.

2. June 1, 1890.

MR. JOHN WILSON,

Bought of WILLIAMS & YOUNG.

45 arithmetics	@	$1.25	.....
97 readers	@	.91	.....
87 grammars	@	.75	.....
59 geographies	@	1.35	.....
83 spellers	@	.45	.....

Ans. $326.77

Show that the bill was paid.

3. Write a receipt for the payment of twenty dollars.

4. $\frac{2}{3}$ of 12; $\frac{2}{3}$ of 24; $\frac{2}{3}$ of 36; $\frac{3}{4}$ of 20; $\frac{3}{4}$ of 24; $\frac{3}{4}$ of 40; $\frac{3}{4}$ of 48; $\frac{4}{5}$ of 20; $\frac{4}{5}$ of 30; $\frac{4}{5}$ of 40; $\frac{4}{5}$ of 60.

5. Reduce 25 bu., 3 pks., 6 qts. to pints.

6. Reduce 5 gallons, 3 quarts, 1 pt., 3 gills to gills.

7. Reduce 5 miles, 6 furlongs, 25 rods to rods.

8. Reduce 12 yards, 2 feet, 11 inches to inches.

L.

1. Write definition of addition; of subtraction; of multiplication; of division.

2. Find the quotient, product, remainder, and sum of 345 and 3105. Mark all the terms.

3. Write the definition of all the terms you have marked.

4. Divide 173245 by 23; 183245÷34; 193245 by 45;

193245 by 56; 193245 by 67; 193245 by 78; 193245 by 89; 193245 by 98.

(Work these by the model in Lesson I.)

LI.

1. Write from dictation $505.875, $3754.75, $49.49, $6801.105, $4975.015, $8705.06.

2. Write a note promising to pay $30 in three months from date.

3. Turn $50.50 into cents; turn 4040 cents into dollars; turn $37.57 into cents; turn 1440 cents into dollars; turn $80 into cents.

4. Denominate numbers have a name; abstract numbers have no name. Arrange these numbers in two columns, placing abstract numbers in one column, denominate numbers in the other: 45, 60, $60, 50 bushels, 85 pints, 95, 84, 10, 10 cents, 189 days, 345 minutes, 345, 606 quarts, 875 pecks, 875, 606, 1, $1, 20, 20 quarts.

5. Divide the following numbers by 8: 871543; 769532; 658432; 48372; 78554. Divide the same numbers by 7.

LII.

1. Divide the following numbers by 87: 331112; 400059; 500032; 10150.

2. $\frac{4}{5}$ of 60; $\frac{3}{4}$ of 48; $\frac{2}{3}$ of 30; $\frac{4}{8}$ of 48. How do you get $\frac{1}{5}$ of a number? $\frac{1}{4}$ of a number? $\frac{1}{3}$ of a number? $\frac{1}{8}$ of a number?

3. How do you get $\frac{1}{5}$ of any thing? How do you get $\frac{1}{3}$ of any thing? How do you get $\frac{1}{4}$ of any thing? How do you get $\frac{1}{8}$ of any thing?

4. If you divide a number by 5, what do you get? If you divide a number by 4, what do you get?

(Teacher should illustrate by dividing some single object, as

an apple, etc., and also by dividing a number of objects and then the numbers on the board.)

5. Write from memory the Table of Dry Measure. Write from memory the Table of Liquid Measure. Write from memory the Table of Long Measure.

LIII.

1. Reduce 3 bushels, 3 pecks, 3 quarts to pints.

(Make the table of unit values as in Lesson XLV., Ex. 4.)

2. Reduce 5 gallons, 3 quarts, 2 pints, 3 gills to gills.

(Bring the measures before the class, and have the pupils use them in measuring water.)

3. Reduce 5 yds. 2 ft. 10 in. to inches.

(Bring the foot rule and yard measure before the class, and use them. This should be done frequently.)

4. Subtract 987 from 6909 until nothing remains.

5. Multiply the following numbers by 20: 23, 45, 56, 87, 95, 96, 65, 47, 38, 27, 35, 42. Multiply the same numbers by 200.

(Teacher will have these worked orally in class.)

6. Divide the following numbers by 97: 18765, 250375, 246956. Prove each example, and mark every term in the division and in the proof.

LIV.

1. Take \$50.50 from \$100 and one mill. Take 10 cents from \$10. Take 9 mills from 9 cents. \$875.375 —\$99.099. Subtract \$375 from \$376.01. Find the difference between forty dollars and forty cents.

2. Which is greater, 5 bushels or 21 pecks? 4 miles or 35 furlongs? 15 gallons or 25 quarts? 8 pecks or

120 pints? 5 yards or 17 feet? 5 yards or 180 inches? 7 miles or 56 furlongs? $50 or 4000 cents?

3. Richmond, June 12, 1890.

MR. WILLIAM WILSON,

Bought of DAY & MOSS.

875 lbs. of lard	@	$ .15	per pound, ...
975 lbs. of butter	@	.36	per pound, ...
784 lbs. of meat	@	.12½	per pound, ...
659 lbs. of sugar	@	.07	per pound, ...
768 lbs. of salt	@	.05	per pound, ...

Ans. $664.78

Show that this bill was paid.

LV.

1. Which is greater, 300 cents or $3.30? 10 miles or 90 furlongs? 10 miles or 3000 rods? 5 pecks or 41 quarts? 2 quarts or 8 pints? 3 feet or 2 yards?

2. Write from dictation the following numbers: $2057.875; $3010.35; $495.175; $90.85; $87.97; $897; $42325.375. Add these numbers.

3. Take 85 cents from one dollar and fifty cents. Take 375 cents from $37.50. Take 75 cents from 75 dollars. Take twenty cents and five mills from 20 dollars.

4. Divide $222.72 by 87; divide $448.50 by 78; divide $432.15 by 67; divide $641.55 by 65; divide $485.52 by 56. Prove each example.

LVI.

1. I gave 46 bushels of wheat for 76 bushels of oats at 23 cents a bushel. How much did the wheat bring me per bushel? *Ans.* 38 cents a bushel.

2. Suppose a field 217 yards square. If I walk around it five times, how many yards will I walk?

Ans. 4340 yards.

3. Suppose you have 800 trees to set out. How many rows will you have, if you put 16 trees in a row? How many trees in a row, if you set 800 trees in 32 rows?

4. How many trees will it require to set out 16 rows, with 800 trees to the row?

5. Are 575 cows at $57.50 each worth more than 5750 sheep at $5.75 each?

6. What year is this? When will you be 55 years old? How many years since 1732? When was the year 1?

7. Write from memory the Table of Dry Measure. Write from memory the Table of Liquid Measure. Write from memory the Table of Long Measure.

LVII.

1. Commit to memory the

TABLE OF TIME MEASURE.

60 seconds	make 1 minute (min.).
60 minutes	make 1 hour (hr.).
24 hours	make 1 day (da.).
7 days	make 1 week (wk.).
365 days	make 1 year (yr.).

2. Repeat the lines, "Thirty days hath September," etc.

3. Write the days of the week. Write the months of the year.

4. Write a note promising to pay $175 in six months, with interest from date.

5. Write an order to your butcher for two different kinds of meat.

6. Reduce 5 hours, 35 minutes, 35 seconds to seconds.

7. Divide $9875.70 by 78; divide $18654.37 by 87. Prove each example.

LVIII.

1. Learn the

TABLE OF AVOIRDUPOIS WEIGHT.

16 ounces	make 1 pound (lb.).
100 pounds	make 1 hundred weight (cwt.).
20 hundred weight	make 1 ton (t.).

For what is this table used?

2. Learn the

TABLE OF TROY WEIGHT.

24 grains	make 1 pennyweight (pwt.).
20 pennyweights	make 1 ounce (oz.).
12 ounces	make 1 pound (lb.).

What is this table used for?

3. Change 15 pounds to ounces. Change 10 tons to pounds. Reduce 25 pounds to ounces. Reduce 15 pounds to ounces. Reduce 15 pounds to pennyweights. Reduce 15 pounds to grains. Reduce 5 cwt. to pounds. Change 85 cwt. to pounds. Reduce 85 cwt. to ounces.

4. Divide 887105 by 23; divide 481096 by 34; 192756 by 45; 871023 by 56; 183051 by 67; 971008 by 78; 1951057 by 89; 63104 by 98.

LIX.

1. Write from memory the Time Table. Write Table of Avoirdupois Weight. Write Table of Troy Weight. Write Table of Liquid Measure. Write for what each table is used.

2. $50 from $80.01; 45 cents from $41; find difference between $70.99 and $80; take $.37 and five mills from $5; $87515.375 from $90000; take $7095.095 from $8000.10.

3. Change $57.57 to cents. Change 4325 cents to dollars. Change $5.50 to cents.

4. Show that $459 \times 362 \times 468$ will equal $724 \times 234 \times 459$.

5. Divide the following numbers by 8: 890756, 3701895, 6930505, 8765434, 673215.

6. $\frac{3}{8}$ of 30; $\frac{5}{6}$ of 60; $\frac{5}{8}$ of 72; $\frac{8}{9}$ of 72.

LX.

1. Write from memory the Table of Avoirdupois Weight. Write from memory the Table of Troy Weight.

2. Reduce 3 tons, 18 cwt. to cwt. Reduce 3 tons, 14 cwt., 37 lbs. to lbs. Reduce 3 tons, 12 cwt., 45 lbs., 13 oz. to ounces.

3. Reduce 5 lbs., 10 oz., Troy Weight, to ounces. Reduce 5 lbs., 9 oz., 16 pwts., to pennyweights. Reduce 5 lbs., 9 oz., 14 pwts., 10 grs. to grains.

4. $\frac{5}{8}$ of 56; $\frac{3}{4}$ of 48; $\frac{9}{10}$ of 80; $\frac{5}{7}$ of 63; $\frac{5}{6}$ of 72.

5. Change $50 to cents; 50 cents to dimes; 500 dimes to dollars; 500 mills to cents.

6. Divide $43602.58 by 89; divide $43602.58 by 98. Prove each example and mark all the terms.

LXI.

1. If 45 yards of cloth cost $303.75, what will 90 yards cost? *Ans.* $607.50.

2. Which is more valuable, 57 horses at $90.75 each or 153 cows at $44.50 each? and how much?

Ans. The cows, $1635.75.

3. $57695+439805+74569+8954+89555+687546+956956=?$

4. $712518+695432+658543+958564+8769543+6895843+9558914=?$

5. Subtract 69857 from 558856 until nothing remains.

6. If I buy 10 oranges at the rate of 5 for 8 cents, and sell them at the rate of 2 for 4 cents, do I make or lose? and how much?

Ans. Make 4 cents profit.

LXII.

1. If 12 lambs cost 30 dollars, at how much must they be sold a head to clear 18 dollars on the cost?

2. What is the value of 35 cases of shoes, each case containing 8 boxes, each box containing 2 dozen pairs, at $2.24 per pair. *Ans.* $15052.80.

(Teacher should always require pupils to place in the multiplicand a number of the same denomination required in the answer.)

3. Divide 16777216 by 8, and the quotient by 8, and so on until you get 1 for a quotient.

4. Divide 43046721 by 9, and your quotient by 9, and so on until you get 1 for a quotient.

5. Write from memory the Table of Dry Measure. Write from memory the Table of Liquid Measure. Write from memory the Table of Time Measure.

LXIII.

1. Divide 875256 by 239; divide 671956 by 358; divide 185195 by 467; divide 250657 by 586. Prove each example.

2. Learn the

TABLE OF SQUARE MEASURE.

144 square inches make 1 square foot (sq. ft.).
9 square feet make 1 square yard (sq. yd.).
30¼ square yards make 1 square rod or pole (sq. rd. or pole).
160 square rods make 1 acre.

What is this table used for?

3. Write from memory the Table of Avoirdupois Weight.
Write from memory the Table of Troy Weight.
Write from memory the Table of Long Measure.

LXIV.

1. What is reduction?
2. What is a denominate number?
3. What is a compound number?
4. Make an example in reduction.
5. Write five denominate numbers.
6. Write five compound numbers.
7. Write the definition of addition; of subtraction; of multiplication; of division.
8. Name and define all the terms of division.
Make an example in division, and mark the terms.
9. Name and define the terms in multiplication.
Make an example in multiplication, and mark the terms.
10. Name and define the terms of subtraction.
Make an example in subtraction and mark all the terms.
11. What is the answer called in addition?

LXV.

1. $\frac{2}{3}$ of 24; $\frac{3}{5}$ of 40; $\frac{5}{6}$ of 42; $\frac{5}{8}$ of 64; $\frac{7}{9}$ of 63.
2. Divide 189355 by 234; divide 189056 by 345; divide 87114 by 456.
3. Divide 193944 by 567. Prove by first method—that is, multiply the quotient by the divisor. Now prove by second method—that is, divide 193944 by your quotient, and observe whether you get 30 for remainder in each division.

4. Multiply the following numbers by 30: 25, 27, 38, 45, 56, 89, 125, 226, 375, 478.

Multiply the same numbers by 300.

5. NASHVILLE, TENN., June 1, 1890.

MR. JOHN BROWN,

Bought of ROSS & MORRIS.

5 bales cotton, each 475 lbs., @ .07 per pound.
8 bales cotton, each 534 lbs., @ .07½ per pound.
9 bales cotton, each 874 lbs., @ .08 per pound.

Ans. $1115.93.

LXVI.

1. Repeat and write the Table of Long Measure.
   Repeat and write the Table of Avoirdupois Weight.
   Repeat and write the Table of Troy Weight.
   Repeat and write the Table of Time Measure.
   Repeat and write the Table of Square Measure.
2. Which is greater, 250 minutes or 4 hours?
   Which is greater, 95 ounces Troy Weight or 6 pounds? 320 poles or 2 acres? 5 acres or 900 poles? 4901 cents or $50.99?
3. Change 20 tons, 16 cwt., 12 lbs. to pounds.
   Change 17 cwt., 85 lbs., 10 oz. to ounces.
4. Reduce 45 bushels, 4 quarts to pints.
   Change $53.37 to cents; 460 cents to dollasr; change 560 dollars to eagles.
5. Divide 235094 by 657. Prove by first method and by second method. Mark all the terms.
6. What is the first method of proving division?
   What is the second method of proving division?
7. If a pint of water weighs a pound, how many pounds in 1 gallon?

LXVII.

1. Divide 350184 by 678. Prove by both methods.
2. Repeat and write the Table of Square Measure. Cut 20 square inches out of paper and carry them to class. Place them so that they will cover a space 20 inches long and 1 inch wide. Place them to cover a space 10 inches long and 2 inches wide. Place them to cover a space 5 inches long and 4 inches wide.
3. Measure a space on the floor 30 inches long and 16 inches wide, and mark it off in square inches.
4. Measure a string a rod long. Then cut it into parts a yard long. Cut each yard into parts a foot long. Cut 1 foot into pieces of 1 inch. Then place 12 of these string inches and 12 paper square inches side by side for comparison.
5. Divide 351094 by 789. Prove by second method, and mark all the terms.

LXVIII.

1. Repeat and write the

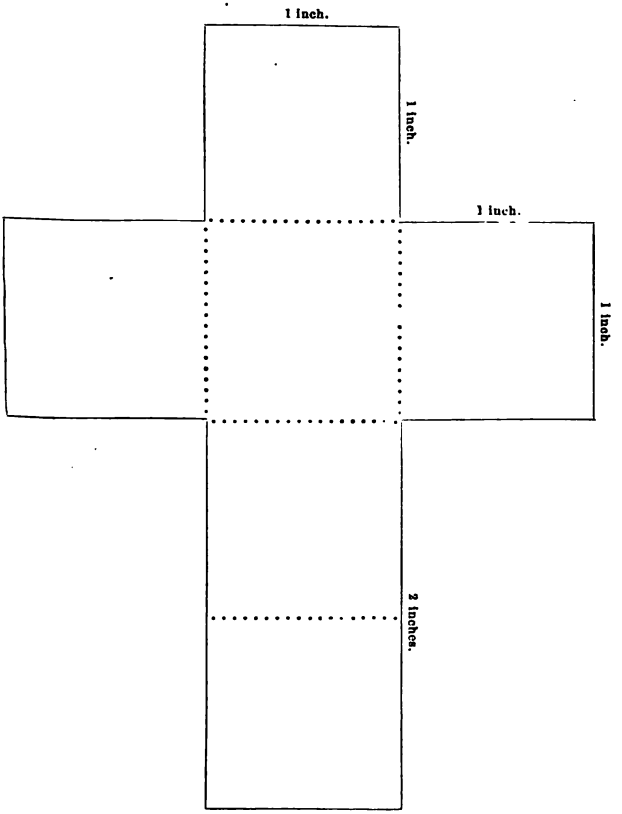
TABLE OF SOLID OR CUBIC MEASURE.

1728 cubic inches make 1 foot (cu. ft.).
27 cubic feet make 1 yard (cu. yd.).

Write what this table is used for.

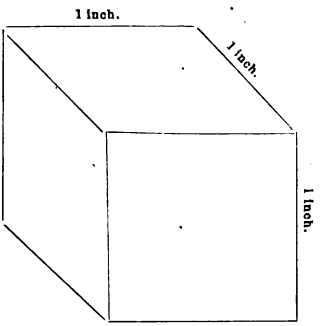
2. Suppose the distance from your home to school be 2 miles, and suppose you carry your dinner with you, how many miles will you walk in going to school 200 days?
3. What do 1 year, 1 day, 1 hour, 1 minute, 1 second equal, if arranged as in Lesson XLV., Ex. 4?

Require each child to bring 12 pieces of pasteboard cut like this figure.



With a sharp knife cut partially through at dotted lines. The teacher, having provided a bottle of mucilage, will show the pupils how to fold the cards and secure them in shape with mucilage and thin paper

to represent cubic inches. The paper will then have this appearance:



4. Measure a string a rod in length, as in Lesson LXVII., and cut as there directed. Provide square inches in paper. Then place the inch strings, square inches, and cubic inches side by side for comparison.

LXIX.

1. Divide 271098 by 789; prove by second method. Divide 380198 by 897; prove by second method. Divide 4701956 by 987, and prove by second method. Divide 7695463 by 978, and prove by second method.

2. The teacher will repeat the instruction suggested in Lesson LXVII. Construct cubic feet and place foot strings, square feet, and cubic feet for comparison.

LXX.

1. Write from dictation: $18075.75; $3703.271; $2709.56; $10101.011; $33003.303; $404.40; $9090.09; $90090.009; $5050.505.

   Add these numbers and prove.

2. Write from dictation 505050, 910035, 6700070,

3500355, 876500, 3033003, 1100110, 9009909, 7077077. Add these numbers, and prove.

3. Take $45.75 from 80 dollars. Take 29 cents from $2. Take 10 dollars from 11 dollars and 1 mill. Take 36 dollars and 75 cents from 90 dollars and 9 mills. Take 37 cents and 5 mills from 65 cents.

4. Divide $18765.75 by 687; by 787; by 896; by by 988.

(The teacher will now review from Lesson L. Completing the review may be considered the close of one hundred days' work.)

LXXI.

1. What is reduction?

2. What is a denominate number? Write five denominate numbers.

3. What is a compound number? Write five compound numbers.

4. Change 5 bu., 3 pks., 4 quarts to pints.

5. Reduce 5 weeks, 4 days, 23 hours to hours.

6. Change 5 yds., 2 feet, 10 inches to inches.

7. Reduce 1 cubic yard to inches.

8. Change 8 square yards to square inches.

9. Reduce 1 ton to ounces.

10. Reduce 6 miles to rods.

LXXII.

1. $\frac{5}{9}$ of 90; $\frac{7}{8}$ of 80; $\frac{5}{6}$ of 72; $\frac{5}{8}$ of 72; $\frac{6}{7}$ of 63.

2. If 12 pencils cost 96 cents, what will 5 pencils cost? Write analysis. Use pencils and cents.

3. If 8 dozen eggs cost 96 cents, what will $1\frac{1}{2}$ dozen cost. Write analysis.

4. Measure the top of a desk or table and tell how many square inches in it.

5. Let each child bring 12 square inches cut from

pasteboard, and cover different spaces on the floor, and require them to calculate the space covered in inches. Do the same with square feet cut from paper. Place on the wall, where they can remain undisturbed, a number of square inches, square feet, and a square yard, all of paper.

6. Divide 3945096 by 987. Prove by first method, and mark all the terms.

LXXIII.

1. What is the value of 2 pounds of quinine at 25 cents an ounce? *Ans.* $8.

2. What is the value of 5 tons of iron at 12½ cents a pound? *Ans.* $1250.

3. What is the value of 5 bushels of chestnuts at 5 cents a quart? *Ans.* $8.

4. Write the Table of Liquid Measure.

5. What is the value of 1 hogshead of molasses at 12 cents a pint? *Ans.* $60.48.

6. How many minutes in 2 weeks?

Ans. 20160 minutes.

7. How many rods in 25 miles? *Ans.* 8000 rods.

Measure a rod on a string, and cut it into yards and feet.

8. Show that $875 \times 384 \times 627$ will equal $1254 \times 875 \times 192$.

LXXIV.

1. Cut a circle of pasteboard, measure its circumference, and show by actual movement how many times it will roll round in moving along a line 12 feet. The circle should be 8 inches in diameter.

2. How many times will a carriage wheel 15 feet in circumference turn around in going 300 yards?

Ans. 60 times.

3. Get a brick, and measure its length and width, and let some child mark one of its greatest surfaces in square inches. A brick is usually 8 inches long and 4 inches wide.

4. A walk is 15 feet long and 6 feet wide; how many bricks will the pavement on it require?

Ans. 405 bricks.

Require the children to mark off in the yard a walk 6 feet wide and 15 feet long. Let them lay down a brick and mark the space it covers; and continue to lay the brick until they have marked off the whole walk in spaces a brick will cover.

5. How many cubic inches in a cubic yard?

Ans. 46656 cu. in.

LXXV.

1. What does *ascend* mean? What does *descend* mean? If reduction is changing a number from one denomination to another, what is *reduction descending?* What is *reduction ascending?*

2. Tell which is reduction ascending, and which is reduction descending, in the following: From 5 dollars to cents; from 3 feet to inches; from 4 feet to yards; from 2 miles to rods; from 20 dollars to eagles; from 3000 pounds to tons; from 36000 minutes to days.

3. In what kind of reduction do you divide? In what kind of reduction do you multiply? If you go to a higher denomination, will the number of units become larger or smaller?

4. Write the names of the Elementary Rules.

5. Divide 569054 by 675; by 765; by 879.

6. Write from dictation $40.40; $1040.04; $20400.004;

$80.90; $808.08; $8080.80; $80800.008; $95010.75. Add these numbers, and prove.

LXXVI.

1. Take $5.05 from $10.001; subtract $.405 from $40.05; take $1.009 from $2; take fifty cents from fifty dollars; from $.90 take seventy cents and five mills; take $3.375 from $20.

2. $\frac{5}{8}$ of 56; $\frac{3}{4}$ of 52; $\frac{5}{6}$ of 48; $\frac{3}{7}$ of 49; $\frac{7}{12}$ of 72.

3. Write the Table of Liquid Measure; write the Table of Long Measure; write the Table of Cubic Measure; Write the Table of Avoirdupois Weight; write the Table of Troy Weight; write the Table of Square Measure; write the Table of Time Measure.

4. How long is it from 6 A.M. to 9 P.M.?

5. The following years were in what centuries: 1825, 1776, 504, 1375, 1400, 601, 1010?

6. Reduce 1 hogshead to gills; reduce 1 mile to rods; reduce 1 ton to ounces; change 1 lb. Troy Weight to grains; change 1 week to seconds. Are these examples of *reduction ascending*, or *descending?*

LXXVII.

1. Change 5 lbs. (Av.), 14 ounces to ounces.
2. Change 5 miles, 7 furlongs to rods.
3. Change 25 yards to inches.
4. Change 25 weeks to hours.
5. Change 3 hhds., 25 gal., 3 qts. to pints.
6. Reduce 25 cu. yds., 26 cu. ft., 875 cu. in. to cubic inches.
7. Divide 3700256 by 769; by 897. Prove each division by second method.
8. What is the value of 25 bushels of nuts at 10 cents a quart? *Ans.* $80.

9. If one mule is worth $98.75, what would 875 mules be worth? *Ans.* $86406.25.

10. If 586 mules are worth $50835.50, what is one mule worth? *Ans.* $86.75.

LXXVIII.

1. Procure a gill, a pint, a quart, and a gallon measure, and practice measuring with these measures before the class.

2. Mark off five distances with points—on the wall or floor—and have them measured first with a foot-rule, and then with a yard-stick.

3. Mark five large spaces on the wall or floor, and have the pupils take length and breadth and calculate the area.

4. Cut a pasteboard as long and wide as a brick. Mark a space on the floor 8 ft. by 6 ft., and lay it off in spaces the size of the pasteboard.

5. Procure several boxes; have the pupils take their length, width, and depth, and find contents in inches.

6. Measure sawdust with peck, quart, and pint measures.

7. Divide 87654 by 203; by 304; by 405; by 506; by 607; by 708; by 809; by 9 8.

LXXIX.

1. New Orleans, June 1, 1890.

Mr. William Ashton,

Bought of Jones & Davis.

578 hhds. of tobacco, each weighing 1870 lbs., @ 8 cents a pound.
943 hhds. of tobacco, each weighing 1563 lbs., @ 7 cents a pound.
765 hhds. of tobacco, each weighing 1764 lbs., @ 9 cents a pound.

Ans. $311093.83.

Show that the bill was paid.

2. Write a receipt for the payment of 400 dollars.

3. Which is of greater value, 5 tons of iron at 5 cents a pound, or 5 hhds. of molasses at 22 cents a quart? *Ans.* The iron is worth $222.80 more.

4. Make a note of $400, payable in six months, with interest.

(Show your pupils note blanks used by business men.)

LXXX.

1. In what kind of reduction do you multiply? In what kind of reduction do you divide?

2. In going from pints to bushels, do you divide or multiply?

3. In changing weeks to hours, do you multiply or divide?

4. In reducing pounds to tons, do you divide or multiply?

(The teacher should give a great many questions like the above. Place a list of such questions on the board.)

5. Reduce 4 tons, 4 cwt., 4 lbs., 4 oz. to ounces.

6. How many square yards in the floor of a room 9 yards long and 4 yards wide?

7. How many square yards in the floor of a room 36 feet long and 15 feet wide?

8. How many bottles, each holding 3 pints, can be filled from 6 gallons of wine? *Ans.* 16 bottles.

(Teacher will use the measures in actual practice.)

9. How many minutes in the spring months?
Ans. 132480 minutes.

10. How many bushels of corn worth 68 cents a bushel will pay for 136 bushels of wheat at 90 cents a bushel? *Ans.* 180 bushels.

11. If you pay 15 cents a dozen for 2 dozen eggs,

and break 9 eggs, for what must you sell the remainder of the eggs so that you may lose nothing?

Ans. 2 cents each.

LXXXI.

1. $\frac{5}{7}$ of 70; $\frac{10}{12}$ of 72; $\frac{3}{10}$ of 90; $\frac{7}{9}$ of 81; $\frac{8}{9}$ of 54; $\frac{3}{4}$ of 72.

2. Divide 47699 by 23; by 34; by 45; by 56; by 67; by 78; by 89; by 98.

3. Take \$90.90 from 100 dollars. Take 10 cents and 8 mills from a quarter of a dollar. Take 65 cents and 1 mill from 10 dollars.

4. Multiply orally in class the following numbers first by 40 and then by 400: 35, 47, 58, 97, 91, 125, 375, 801, 101, 202, 50.

5. Subtract 937 from 8883 until nothing remains.

6. Write the lines: "Thirty days hath September," etc.

7. How many seconds in the month of September?

8. If you buy 12 apples at 2 for a cent, and sell them at 3 for a cent, do you gain or lose, and how much?

9. Turkeys are worth one dollar; ducks, 75 cents; chickens, 50 cents. How many of each can you buy for \$20.25, if you buy the same number of each?

LXXXII.

1. How many cubic inches in a box 8 inches long, 6 inches wide, and 3 inches deep? Make the box of pasteboard, and fill it with cubic inches of paper.

2. How many cubic feet in a cellar 10 feet wide, 15 feet long, and 6 feet deep? Draw a figure of the cellar.

3. How many cubic inches in a brick?

(Bring before the class a real brick, a wooden brick, and another wooden brick cut into cubic inches.)

4. How many cubic inches in a cubic foot? How many cubic inches in 1 cubic yard?

5. What is a denominate number? a compound number? Write five of each.

6. Which is a higher denomination, a mile or a rod? a day or an hour? pounds or ounces? minutes or weeks?

7. Would you multiply or divide in reduction ascending? What is reduction ascending? What is reduction descending?

8. Write the names of the Elementary Rules, and tell what each is used for.

9. Find the sum, product, difference, and quotient of 789 and 5524, and mark all the terms.

LXXXIII.

1. Prove that 8 bu., 3 pks. equal 35 pecks. Prove that 35 pecks equal 8 bu., 3 pks.

Proof. 8 bu. 3 pks.
4 pks.
35 pks.

Proof. 4)35 pks.
8 bu. + 3 pks.

2. Change 5 bu., 2 pks., 4 qts. to quarts.

5 bu. 2 pks. 4 qts.
4 pks.
22 pks.
8 qts.
180 qts.

Proof. 8)180 qts.
4)22 pks. + 4 qts.
5 bu. + 2 pks.

3. Prove that 5 lbs., 10 ozs. equal 90 ounces. Show that 90 ounces equal 5 lbs., 10 oz. Which of these

examples would you call reduction ascending? Which reduction descending?

4. Reduce 15 lbs., 10 oz., 6 pwt. to pennyweights. Change 3806 pwts. to higher denominations. Repeat and write Table of Troy Weight.

5. Divide 9576 by 230; by 340; by 450; by 560; by 670; by 780; by 890; by 980.

LXXXIV.

1. Repeat the Time Table.
   How many weeks and days in 76 days?
   How many units make a score? 5 score?
   Place a score of objects before your class.
2. How many dozen in 3 score? *Ans.* 5 dozen.
   How many score in 5 dozen? *Ans.* 3 score.
3. What is the value of 6 gallons of oil at 6 cents a pint? *Ans.* $4.08.
4. Reduce 6 gallons to gills.
5. Change 1200 pints to higher denominations. Prove.
6. 5 is $\frac{1}{5}$ of what number? 6 is $\frac{1}{7}$ of what number? 8 is $\frac{1}{9}$ of what number? 10 is $\frac{1}{10}$ of what number? 7 is $\frac{1}{8}$ of what number? 12 is $\frac{1}{7}$ of what number?
7. $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 16; $\frac{4}{5}$ of 20; $\frac{5}{6}$ of 30; $\frac{6}{7}$ of 42.

LXXXV.

1. Reduce 1400 inches to higher denomination. Prove as in Lesson LXXIV.
2. Reduce 1560 ounces to higher denominations. Prove.
3. Reduce 1485 rods to higher denominations. Prove.
4. Change 1575 sq. rods to acres. Prove.

5. The quotient is 875, the divisor 35, remainder 34; what is the dividend? Arrange as below:

Divisor. Dividend. Quotient.

35) ..... (875

.....

.....

.....

.....

.....

34 Remainder.

875 Quotient.
35 Divisor.
4375
2625
34 Remainder.
30659

30659 is dividend.

6. The subtrahend is 8954, the remainder is 989; what is the minuend? Arrange as below:

..... Minuend.
8954 Subtrahend.
989 Remainder.

Minuend 9943. Prove.

Place sum of subtrahend and remainder above instead of below, as is usually done.

7. The product is 80495, the multiplier 85; what is the multiplicand? Arrange as below:

... Multiplicand.
85 Multiplier.
....
....
80495 Product.

85) 80495 (947
765
399
340
595
595

947 is Multiplicand. Proof: $947 \times 85 = 80495$.

LXXXVI.

1. How many lemons at 3 cents each will pay for 4 boxes of oranges, each containing 150 oranges, at 6 cents for 1 orange? *Ans.* 1200 lemons.

2. I bought 200 acres of land at $40 an acre. I sold one-half of the land at $50 an acre and the other half at $30 an acre; did I gain or lose?

3. ATLANTA, Sept. 1, 1890.

MR. JAMES ROE,

Bought of KLINE & HARRIS.

8 boxes of oranges, each containing 144 oranges, @ $0.025 each.
14 boxes of oranges, each containing 144 oranges, @ .03 each.
37 boxes of oranges, each containing 144 oranges, @ .04 each.
29 boxes of oranges, each containing 144 oranges, @ .035 each.

Ans. $448.56.

Show that the bill was paid.

4. Write from dictation: $0.01, $.035, $.209, $.156, $.25, $.303, $.49, $.162, $.20, $.301. Add and prove.

5. $60.60—$39.59; take one dollar and fifty-seven cents from $2; take ten cents from ten dollars; take 14 cents and 1 mill from $20.

LXXXVII.

1. Write from memory the Table of Dry Measure.

2. Write from memory the Table of Liquid Measure.

3. Write from memory the Table of Long Measure.

4. Change 1500 quarts to higher denominations. Prove.

5. Reduce 1575 gills to gallons. Prove.

6. Change 1250 inches to higher denominations. Prove.

(The teacher should not fail to have the proof performed in every example.)

7. Change 40000 cubic inches to higher denominations. Prove.

8. Write the Table of Cubic Measure.

9. What is a cube? Make one. Draw a likeness of a cube.

10. Change 50000 minutes to weeks. Prove.

LXXXVIII.

1. Change 3853 inches to higher denominations. Work this example by placing quotients over the dividend, instead of under it, thus:

107 yds.
3 ft.|321 ft.+1 in.
12 in.|3853 inches.

yds.	ft.	in.
107	0	1
	3 ft.	
	321 ft.	

Proof: 12
3853 inches.

2. Change, as above, 47533 pounds to higher denominations. Prove.

3. Change 3 tons, 16 cwt., 70 lbs., 10 ounces to ounces. Prove.

4. Divide 37845 by 908; by 807; by 706; by 605; by 504; by 403; by 302; by 203.

LVXXXIX.

1. Make an example in subtraction; one in multiplication; one in division. Mark all the terms.

2. Divide 37845 by 980; by 870; by 760; by 650; by 540; by 430.

3. An estate was divided: each daughter received 1358 dollars; each son twice as much as a daughter; the widow as much as all her children; there were 4 daughters and 3 sons; what was the widow's share?

Ans. $13580.

4. Change 1275 rods to miles. Change 1285 miles to rods.

5. Reduce 18375 sq. yds. to sq. inches. Change 18375 sq. in. to sq. yds.

6. Reduce 5043 ounces to higher denominations. Change 5043 cwt. to lower denominations.

7. A cord of wood is 8 feet long, 4 feet wide, and 4 feet high. How many cubic feet in 2 cords?

Ans. 256.

8. How many cubic feet in a pile of wood 38 feet long, 5 feet high, and 4 feet wide?

9. How many feet in a cord of wood? How many cords in the pile of wood in the eighth example?

Ans. 5 cds. 120 cu. ft.

XC.

1. NASHVILLE, May 1, 1891.

Mr. JOHN DOE,

Bought of BROWN & ROSS.

37 sacks of salt, each weighing 256 pounds, @ 3½ cents a pound.
59 sacks of salt, each weighing 248 pounds, @ 4½ cents a pound.
28 sacks of salt, each weighing 325 pounds, @ 5½ cents a pound.
32 sacks of salt, each weighing 485 pounds, @ 2½ cents a pound.

Ans. $1878.46.

Brown & Ross agreed to take cloth at 40 cents per yard for the bill. How many yards did Mr. Doe give them? *Ans.* 4696 yds.

2. How many mills in a cent? How many mills in a half-cent?

3. Change 5375 cents to dollars.

4. Change 3½ hours to minutes. Change 3½ yards to inches.

5. Change 4½ miles to rods. Change 10½ gallons to pints.

6. What is reduction? What is reduction descending? What is reduction ascending? Make an example in reduction descending. Make an example in reduction ascending.

XCI.

1. Make an example in Troy Weight; work it and prove it.

2. Make an example in Time Table; work it and prove it.

3. Make an example in Avoirdupois Weight; work it and prove it.

4. Make an example in Long Measure; work it and prove it.

5. Make an example in Square Measure; work it and prove it.

6. Make an example in Cubic Measure; work it and prove it.

7. Make a bill of sale of school-books.

8. Write a receipt for the payment of $580.25.

9. Write a note promising to pay 760 dollars in three months, with interest.

10. Write what a denominate number is.

11. Write what a compound number is.

12. Write five denominate numbers; five compound numbers.

13. Show your teacher that you can write three and one-half cents; two and $\frac{1}{2}$ cents; 4 and one-half cents.

XCII.

1. 12 is $\frac{1}{5}$ of what number? 15 is $\frac{1}{6}$ of what number?
16 is $\frac{1}{8}$ of what number? 14 is $\frac{1}{9}$ of what number?
18 is $\frac{1}{7}$ of what number? 18 is $\frac{1}{8}$ of what number?

(Work these examples with chalk rings or with objects.)

2. $\frac{5}{7}$ of 49; $\frac{6}{7}$ of 42; $\frac{7}{8}$ of 80; $\frac{5}{12}$ of 84; $\frac{5}{7}$ of 84.

3. Reduce 5000 lbs. to tons. Prove.

4. Change 36000 minutes to higher denominations. Prove.

5. Change 4 bu., 3 pks., 7 qts., 1 pt. to pints. Prove.

6. How many cubic feet in a cellar 16 ft. long, 12 ft. wide, 6 ft. deep? How many cubic yards in the same cellar?

7. Divide 1870583 by 23; by 234; by 2345. Mark all the terms.

XCIII.

1. How do you get the surface in inches of the top of a box?

How do you get the cubic contents of the box?

(Teacher will procure a box and have children measure it.)

2. How long, wide, and deep is a cord of wood?

How do you proceed to find the cubic feet in a cord of wood?

3. How many cubic inches in a stone step 7 feet long, 4 feet wide, and one foot thick?

4. How many cubic feet in a pile of wood 20 feet long, 8 feet wide, 6 feet high? How many cords in the same pile?

Make a cubic inch and a cubic foot of pasteboard.

5. Find the value of a train load of coal consisting of 17 cars, 425 bushels to each car, at 12½ cents per bushel. *Ans.* $903.12½.

6. Divide 137085 by 34; by 345; by 3456.

7. Four boys commenced to play with 200 marbles; when they stopped playing one had 50, having won ten; one had 60, having won 12; one had 30, having neither won nor lost; what did the fourth boy have when they commenced to play?

XCIV.

1. How often will a pasteboard circle 15 inches in diameter turn round in rolling along a line 15 feet? (Use the circle.)

2. How many bricks will cover a pavement 75 feet long and 8 feet wide? *Ans.* 2700.

3. Reduce 27 cu. yds., 15 cu. ft., 500 cu. in. to cubic inches. Prove.

4. Reduce 840 sq. in. to square yards. Prove.

5. How many rods in $1\frac{1}{2}$ miles?
Lay off in the school-yard a line one rod long.
Lay off a square rod.

6. How many acres in 1800 square rods?

7. Take \$30.10 from \$100; subtract 50 cents, 7 mills from \$5; take $37\frac{1}{2}$ cents from $2\frac{1}{2}$ dollars.

8. Divide 268519 by 45; by 456; by 4567.

XCV.

1. Square measure is used to measure surfaces having length and width. As a unit of measure we use one square inch, or one square foot, or one square yard, or one square rod. Cut a number of square inches, square feet, and square yards out of paper—the more the better—and place them on measured surfaces.

2. Cubic measure is used to measure solids, or objects and space having length, breadth, and thickness. The unit of measure is a cubic inch, or a cubic foot, or a cubic yard. You should make a cubic foot and cubic inch out of pasteboard, and find a box that contains a cubic yard.

3. If 5 yards of cloth cost \$9.25, what will 37 yards cost? *Ans.* \$68.45.

4. If 15 yds. of cloth cost $33.75, what will 375 yds. cost? *Ans.* $843.75.

5. If 875 bushels of wheat are worth $831.25, what is the value of 536 bushels? *Ans.* $509.20.

6. What is the value of 3 car loads of shoes, each car containing 75 cases, each case 12 boxes, each box 2 dozen pairs at $2.50 a pair? *Ans.* $1620.00.

(Teacher should be particular in this example, and in all others, to have the multiplicand like the required answer.)

XCVI.

1. What is reduction? Write definition of reduction descending. Write definition of reduction ascending.

2. Reduce 5800 qts. to higher denominations. Prove.

3. Reduce 3 lbs., 10 oz., 15 pwt., 10 grs. to grains. Prove.

4. Change 5000 gills to higher denominations. Prove.

5. Change 10 hhds., 25 gals., 3 qts., 1 pt. to pints. Prove.

(The teacher should require in these examples, as well as in all others, the proof to be worked without fail.)

6. Divide 376958 by 5; by 56; by 567; by 5678.

7. Write from memory the Time Table; the Table of Liquid Measure; the Table of Dry Measure; the Table of Long Measure; the Table of Square Measure; the Table of Cubic Measure; the Table of Avoirdupois Weight; the Table of Troy Weight; the Table of United States money.

XCVII.

1. Write an order to your butcher. Write an order to your grocer. Write a receipt for the payment of $37.50.

(The teacher should instruct children in making promises to pay money in the future—or promissory notes. They should exhibit blank checks and instruct how money is taken out of banks in which it has been deposited for safe-keeping.)

2. Divide 365000 by 6; by 67; by 678; by 6789.

3. 10 is $\frac{1}{2}$ of what number? 11 is $\frac{1}{3}$ of what number? 12 is $\frac{1}{4}$ of what number? 13 is $\frac{1}{5}$ of what number? 5 is $\frac{1}{6}$ of what number? 8 is $\frac{1}{7}$ of what number? 9 is $\frac{1}{8}$ of what number? 7 is $\frac{1}{8}$ of what number?

(Use chalk rings on the board or your wire.)

4. $\frac{3}{5}$ of 75; $\frac{4}{7}$ of 56; $\frac{5}{9}$ of 81; $\frac{7}{12}$ of 84; $\frac{3}{7}$ of 21; $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 12; $\frac{5}{8}$ of 24.

5. Multiply the following numbers by 300: 75, 83, 61, 79, 97, 197, 279, 361, 483, 756. Multiply the same numbers by 30.

6. What is the value of a car load of 35 barrels of beans, each barrel holding 2 bushels, at $12\frac{1}{2}$ cents a pint? *Ans.* $560.

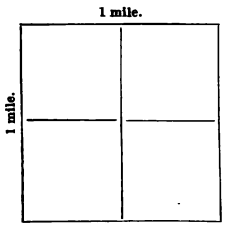
XCVIII.

1. Divide 183148 by 7; by 78; by 789; by 7898.

2. A boy boards with me and goes to school; he is to pays me three dollars a week board and $5 a month for tuition. What will he owe me at the end of ten months? *Ans.* $170.

3. Get a closed box; measure its surfaces before the class, and have the children calculate the area of the surfaces.

4. Find the number of seconds in the summer months.



5. This field is 1 mile square. I wish to make four square fields; what will the outside fences and cross fences cost at $1.50 a rod?

Ans. $2880.

6. How many rings can you make from 1 lb. of gold if you put 12 pwts. in each ring? *Ans.* 20 rings.

XCIX.

1. What is a coin? Name different kinds of coins and what they are made of. Where are they made?

2. Turn 523 cents into dollars by putting a point between the "5" and the "2." Turn 3500 mills into dollars by putting a point between the "3" and the "5."

3. Add 50 dollars, 50 cents, 1500 mills, $87.50, 345 cents, 2425 mills, 5 dollars, 5 cents, 5 mills, 1685 mills.

4. By actual measurement find the area of your school-room floor. Find the area of the top of your teacher's table.

5. Tell how many feet in a string that will reach around your room.

6. By your own measurement find the surface of one side wall and one end wall of your room. If you get the surface or area of the floor, would not that be the same as the area of the ceiling?

7. If you buy a hogshead of wine at $100, and twenty gallons leak out, can you tell what the wine remaining would be worth at $2.50 a gallon? Would you gain or lose?

C.

1. With $480 how many pounds of berries can you buy at 25 cents a pound? *Ans.* 1920 lbs.

2. At 4 cents a pound what will 35,064 lbs. of iron be worth?

3. If postage be 1 cent an ounce, what will it cost to mail one ton of newspapers? *Ans.* $320.

4. How many cubic inches in a marble slab 30 inches long, 20 inches wide, and 6 inches thick?

5. How many cubic inches in a slab 3 feet long, 20 inches wide, 6 inches thick? Can you multiply feet by inches?

6. How do you get the area of a surface? Show what you know about it by finding the area of the platform in your room.

7. How many units make a dozen? How many units make 1 score?

8. Change 7 dozen to scores.

9. We say 365 days make 1 year; but the exact length of a *solar* year is 365 days, 5 hours, 48 minutes, 46 seconds, or 365 days and 6 hours. Then, every fourth year, we add one day, and thus take up the six hours left out for three years; this year we call leap year.

CI.

1. How many years in 1 century? How many minutes in 1 century?

2. Reduce 13 gallons, 1 qt., 1 pt., 3 gills to gills. Prove.

3. Change 1 mile to inches.

4. I bought four loads of hay weighing 2400 pounds, 3750 pounds, 6500 pounds, and 4750 pounds

respectively; how many tons did I buy? How many pounds make 1 ton?

5. How long, wide, and deep is a cord of wood? How many feet in 1 cord?

6. If you sell 40 horses for 900 sheep at $3.50 each, what do you get for each horse? *Ans.* $78.75.

7. Reduce 5 years, 9 months, 3 weeks, 5 days to days. Prove.

8. Reduce 10 gals., 2 qts., 1 pt., 1 gill to gills. Prove.

9. 12 dozen make a gross. How many units in 5 gross?

10. Commit this table to memory:

100 pounds of nails make 1 keg.
196 pounds of flour make 1 barrel.
200 pounds of beef make 1 barrel.
32 pounds of oats make 1 bushel.
60 pounds of wheat make 1 bushel.
56 pounds of corn make 1 bushel.

CII.

1. Divide 293058 by 8; by 89; by 896; by 8965.

2. Reduce 5 bu., 3 pks., 5 qts., 1 pt. to pints. Prove.

3. Reduce 5000 ounces to tons. Prove.

4. Reduce 28800 sq. in. to sq. yds. Prove.

5. Change 1,000 gills to higher denominations. Prove.

6. October 1, 1890.

MRS. BROWN,

Dr. to SMITH & DAVIS.

25 barrels flour,	@	$5.75
462 bushels wheat	@	.95
34 barrels rye	@	4.85
135 bushels oats	@	.65

Show that the bill was paid.

CIII.

1. What is a bill? Make one.

2. What is a receipt?

3. Write a receipt for the payment of $30.40.

4. Write a promissory note in which you promise to pay $50 in six months, with interest at 7 per cent.

5. $987 \times 76 \times 863 = 863 \times 1974 \times 38$.

6. What is the meaning of the word "dividend?" Of "minuend?" Of "multiplicand?"

7. What is the answer called in addition? In subtraction? In multiplication? In division?

8. Find the sum, product, difference, and quotient of 879 and 9669; and mark all the terms.

9. Write from dictation $1.12\frac{1}{2}$; $.376; $29.008; $39.105; $8.87\frac{1}{2}$; $47.06; 9 dollars; 9 dimes; 9 mills. Add and prove.

CIV.

1. Add 57695, 4695, 18095, 875654, 8769, 760543, 87695; and prove, thus:

	57695
	4695
	18095
	875654
	8769
	760543
	87695
First sum	1813146
Second sum	1755451
Proof	57695

Under the sum of all the numbers place the sum of all except the first number; subtract, and the remainder should be equal to first number.

Examples of this kind furnish fine training in concentration, if both sums be carried along together. Thus, adding the first line, we have 5, 8, 17, 21, 26, 31, 36. Place the "1" of 31 under the "6" of 36, as you see in the example. Now, carrying 3, we have

12, 16, 22, 27, 36, 45, 54. Place the "5" of 45 under the "4" of 54. Carrying "5," we have 11, 16, 23, 29, 35, 41. Now observe that you do *not* place the "5" of 35 under "1" of 41, because 54 was the sum of all the previous column, and the second sum of second column exclusive of the upper figure was 45. Hence, to make the second sum in third column, you add "4," and to make first sum of same column, you must add "5," giving you "4" under "1."

2. Add 769, 3647, 3694, 87695, 37695, 28028, 67054. Prove by producing both sums at once.

3. Add 486958, 498329, 673, 69549, 896954, 295487. We hope no pupil will be discouraged with his first trials, for we are certain he will enjoy success when he shall attain it.

CV.

1. 15 is $\frac{1}{5}$ of what number? 17 is $\frac{1}{8}$ of what number?

2. 14 is $\frac{1}{4}$ of what number? 16 is $\frac{1}{8}$ of what number?

3. 18 is $\frac{1}{3}$ of what number? 25 is $\frac{1}{2}$ of what number?

4. 28 is $\frac{1}{2}$ of what number? 35 is $\frac{1}{2}$ of what number?

5. 30 is $\frac{1}{3}$ of what number? 40 is $\frac{1}{4}$ of what number?

6. 20 is $\frac{1}{5}$ of what number? 50 is $\frac{1}{4}$ of what number?

7. 30 is $\frac{1}{4}$ of what number? 40 is $\frac{1}{5}$ of what number?

8. $\frac{2}{3}$ of 6; $\frac{2}{3}$ of 9; $\frac{2}{3}$ of 12; $\frac{2}{3}$ of 18; $\frac{2}{3}$ of 24.

9. $\frac{3}{4}$ of 12; $\frac{3}{4}$ of 16; $\frac{3}{4}$ of 28; $\frac{3}{4}$ of 32; $\frac{3}{4}$ of 36.

10. $\frac{4}{5}$ of 10; $\frac{4}{5}$ of 20; $\frac{4}{5}$ of 25; $\frac{4}{5}$ of 35; $\frac{4}{5}$ of 45.

11. $\frac{5}{6}$ of 12; $\frac{5}{6}$ of 24; $\frac{5}{6}$ of 36; $\frac{5}{6}$ of 42; $\frac{5}{6}$ of 48; $\frac{5}{6}$ of 54.

12. Take \$.875 from \$875. Take 39 cents from 39 dollars. Take 4 dollars and $37\frac{1}{2}$ cents from twenty dollars.

CVI.

1. The divisor is 874, the quotient 976; what is the dividend?

2. The divisor is 874, the dividend 876; what is the quotient?

3. The subtrahend is 3659, the remainder is 4645; what is the minuend?

4. The subtrahend is 4764, the minuend is 4769; what is the remainder?

5. What are the names of the different places in reading numbers?

6. How many figures in a period? Write a number with three full periods.

7. What is a denominate number? What is a compound number?

8. What is reduction? What is reduction ascending? What is reduction descending?

9. Divide 8695101 by 9; by 98; by 987; by 9887.

10. Add 875, 9687, 57695, 37654, 89764, 897234. Try to prove by finding two sums at once, as in Lesson CIV. If you do not succeed at first, you must not be discouraged.

CVII.

1. Reduce 18575 ounces to tons. Prove.

2. Reduce 20 hhds., 35 gals., 3 qts., 1 pt. to gills. Prove.

3. Change 1900 rds. to miles. Prove.

4. Reduce 3500 in. to yards. Prove.

5. Reduce 25 bu., 3 pks., 1 pt. to pints. Prove.

6. I wish to put 1025 bushels of wheat in sacks, each holding 2 bu., 1 pk. If the last sack be not full, how much will be in it? *Ans.* 1 bu., 1 pk.

(The teacher should request each girl to bring three or four little bags four by six inches. Procure, also, a peck of saw dust or sand. Some little measure may be taken to represent a bushel. Let them also see the real sack used in trade.)

7. How many bottles, each holding 1 qt., 1 pt., will be needed to bottle 400 gals. of wine?

(Bring before the class bottles and a bucket of water, and bottle the water.)

CVIII.

1. Measure a cigar-box, and tell how many square inches in its six faces. Tell how many cubic inches in the box.

2. Measure each wall of your room, and tell how many square yards of paper will cover each of the four walls.

Take the measure of each wall in feet. Then lay off paper to represent each wall, each paper being as many inches long and as many inches wide as the wall is long and wide in feet. To get the ceiling, measure the floor. Then let the children suppose the inches to be feet.

Measure the inside of a large cigar-box; find the square inches to cover each inner surface; then cut paper the proper size and place in the box on the surfaces.

3. Measure the length, width, and depth of your room, and tell how many cubic feet of air in the room. Tell how many cubic inches for each scholar to breathe.

4. Bring in a brick, and find how many square inches in its surfaces. Tell how many cubic inches in it.

(Have a wooden brick cut into cubic inches.)

5. How many bricks will lay a pavement 8 feet wide and 10 feet long? *Ans.* 1440.

(Use a paper size of surface of brick.)

6. How long is it from twenty minutes past six A.M. to fifteen minutes past 5 P.M.? What is the meaning of A.M. and P.M.?

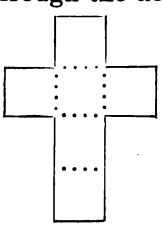
7. How many times will a clock strike in 1 year?

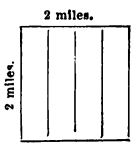
CIX.

1. I wish to ship cigars in boxes 6 in. long, 4 in. wide, 4 in. deep; how many of these boxes can I pack in a large box 3 ft. long, 2 ft. wide, two feet deep? *Ans.* 216.

(Get the large box and fill it with paste-board boxes made by the children.)

2. What is a square? Make one of paper. What is a cube? Make one of pasteboard by cutting it thus, and cut half through the dotted lines:





3. This field is 2 miles square; how many rods of fence will inclose it, and also make four fields?

4. Suppose that every day you set apart 8 hours for sleep, 3 hours for meals, six hours for school, 2 for study, and the rest for pleasure; how much time will you give to your sleep, your meals, your school, your study, your pleasure in one year?

CX.

1. How many square inches of paper will lie on a half square foot? Cut the inches and place them.

2. How many square inches will lie on a square six inches long and six inches wide? Place them.

3. If I go north and you go south from the same point, at the rate of 6 miles an hour, how far apart shall we be in fourteen days if we walk ten hours a day? *Ans.* 1680 miles.

Place a point on the floor; draw two lines from it; put two images on the lines, and show their departure every hour for the first day.

4. How many cubic inches in a cubic foot?

5. How long, wide, and deep is a cord of wood?

6. What is a pile of wood worth 87 ft. long, 12 ft. wide, 8 ft. deep, at $3.75 a cord?

7. A field is 80 rods long, 60 rods wide, with a path 100 rods long diagonally through it; if you go round the field and I go through it, where will we meet if we walk equally fast? Draw the plan of the field.

CXI.

1. Divide 730919 by 9; by 98; by 987; by 9876.

2. Write from memory the Tables of Troy Weight, of Avoirdupois Weight, of Long Measure, of Square Measure, of Cubic Measure, of Time Measure, of Dry Measure, of Liquid Measure.

3. Add 46735, 490875, 67354, 36785, 497634, 876954, 37554 by double sums as in Lesson CIV.

4. 13 is $\frac{1}{7}$ of what number? 15 is $\frac{1}{8}$ of what number? 14 is $\frac{1}{8}$ of what number? 16 is $\frac{1}{8}$ of what number? 12 is $\frac{1}{9}$ of what number? 11 is $\frac{1}{11}$ of what number?

5. $\frac{3}{4}$ of 12; $\frac{4}{5}$ of 20; $\frac{2}{3}$ of 36; $\frac{5}{8}$ of 48; $\frac{6}{7}$ of 49; $\frac{7}{9}$ of 63; $\frac{5}{8}$ of 24; $\frac{5}{8}$ of 40; $\frac{5}{8}$ of 72.

6. Write from memory the definition of reduction. Write the definition of reduction descending. Write the definition of reduction ascending.

CXII.

1. Reduce 3 tons, 15 cwt., 75 lbs., 12 oz. to ounces. Prove. Work by following model and then analyze your work.

ton.	cwt.	lbs.	oz.
3	15	75	12

20 cwt.
75 cwt.
100 lbs.
7575 lbs.
16 oz.
45462
7575
121212 oz.

oz. oz.
16)121212
100 lbs.)7575 lbs.+12 oz.
20 cwt.)75 cwt.+75 lbs.
3 tons+15 cwt.

2. Change 33428 minutes to higher denominations. Prove by model in example 1, and analyze.

3. Reduce 4567 gills to higher denominations. Prove by model in example 1, and analyze.

4. How many spoons can be made from 10 lbs., 10 oz. of silver if each spoon contains 15 pennyweights of silver? *Ans.* 173 spoons and 5 pwts. left.

CXIII.

1. March 1, 1891.

MR. RICHARD ROE,

Bought of BROWN & JONES.

37 bbls. of sugar, each containing 275 lbs., @ 12½ cts. a pound.
58 bbls. of sugar, each containing 325 lbs., @ 11 cts. a pound.
67 bbls. of sugar, each containing 243 lbs., @ 10½ cts. a pound.
38 bbls. of sugar, each containing 284 lbs., @ 12 cts. a pound.

Ans. $6349.92.

Show that the bill was paid.

2. April 3, 1891.

ROSS & MILLER,

Bought of UNION STOCK CO.

87 horses at $88.75 each.
96 mules at $125 each.
63 cows at $51.50 each.
75 Shetland ponies at $75 each.

Ans. $28590.75.

Show that the bill was paid.

3. Write from dictation: $20; twenty cents; 12½ cents; 10 cents and 5 mills; 37½ cents; $1.87½; 40 cents and 9 mills; 5 dollars; ten dollars and ten cents. Add these numbers and prove.

CXIV.

1. What is one cow worth, if you give 58 cows for 29 horses at 88 dollars each. *Ans.* $44.

Analyze.

2. Remove the point from these numbers, and you change them to cents, thus: $5.48=548 cents; $26.05 equals how many cents? $10.15 equals how many cents? $3.62? $51.75? $6.42? $9.20? $81.85? $67.37? $63.10? $47.55? $18.17? $7.55?

3. Change these numbers to dollars by putting points in them, thus: 362 cents equals $3.62; 575

cents? 975 cents? 187 cents? 3750 cents? 2650 cents? 4805 cents? 2085 cents? 3636 cents?

4. Multiply the following numbers by 40 at your desk, and then multiply them orally in class: 46, 77, 89, 189, 225, 375, 845, 909, 203, 667, 101, 111, 27. Multiply the same numbers by 400.

5. Divide 173845 by 23; by 234; by 2345.

CXV.

1. Divide 217384 by 34; by 345; by 3456.

2. Some rich men cover the walls of their rooms with silks and velvets, which must make the rooms very beautiful. How many square yards of velvet would cover the side walls of your school-room?

(Let the pupils do the measuring.)

3. How many square yards of silk would cover the end walls of your room?

4. What would it cost to paint the ceiling of your room in pretty blue paint, at $2.25 a square yard? Remember you can get the surface of the ceiling by measuring the floor.

5. What is the value of 5 bushels of chestnuts at 16 cents a quart? *Ans.* $25.60.

6. *To get the cost of a number of yards, multiply the price of one yard by the number of yards.* Make an example under this rule; work it, and analyze.

7. *Divide the whole cost by the number of yards, to get the price of one yard.* Make an example under this rule; work it, and analyze.

CXVI.

1. To get the cost of a number of pounds, multiply the price of one pound by the number of pounds.

Make an example under this principle; work it, and analyze.

2. Divide the whole cost by the number of pounds to get the price of one pound. Make an example under this principle; work it, and analyze.

3. If 5 hats cost $45, what will 36 hats cost?

4. If 58 horses cost $11600, what will 37 horses cost? Analyze both examples.

5. Make two examples similar to the third and fourth examples.

6. Make a bill of purchase of five articles sold in book-store. Work it, and read it before your class.

CXVII.

1. $568 \times 344 \times 189 = 1136 \times 172 \times 189$.

2. Write an order on your butcher for three different kinds of meat, naming the number of pounds you wish of each.

3. What is the value of 5 gallons of sawdust at 2 cents a pint? Get the sawdust, measure it, and let the pupils handle it in working the example.

4. What is the value of 5 bushels of strawberries at 5 cents a pint? *Ans.* $16.

5. I went to New York, having $1,000 when I started. I was gone three weeks. I traveled 2,000 miles at 3 cents a mile, and spent $3.50 a day for my board. On my return home I lost my valise and some money. I had $225 when I reached home. How much money was in the valise? *Ans.* $641.50.

6. Divide 327549 by 45; by 456; by 4568.

CXVIII.

1. Make an example in Troy Weight in reduction descending. Prove your work.

2. Make an example in Time Measure in reduction ascending. Prove your work.

3. Make an example in Avoirdupois Weight in reduction descending. Prove.

4. Make an example in Long Measure in reduction descending. Prove.

5. Make an example in Square Measure in reduction ascending. Prove.

6. Make an example in Cubic Measure in reduction descending. Prove.

7. Make an example in Liquid Measure in reduction ascending. Prove.

CXIX.

1. Divide the following numbers by 10; by 100: 87505; 965325; 461102; 376543. Divide the same numbers by 11. Divide the same numbers by 12. Do this work by short division.

2. Cut a circle of pasteboard, measure the circumference of it; measure a line on the floor, calculate the number of times the circle will turn on the line. Then prove by rolling the circle along the line.

3. Add 87654, 90973, 75694, 55672, 876, 95876, 47895. Prove by finding two sums at once, as in Lesson CIV.

4. A stranger bought of Mr. Brown a hat worth five dollars, and handed him a fifty-dollar note. Mr. Brown had to go to a bank for change. Returning, he gave $45 to the stranger, whom he never saw again. The note was counterfeit, and Mr. Brown had to repay the bank good money. How much did he lose?

CXX.

1. Make an example where you give 59 bushels of corn for a certain number of bushels of oats at a stated price per bushel, and ask the price of the corn per bushel. Work it.

2. Make an example in which you wish to know the difference in minutes between spring and summer. Work it and explain.

3. Reduce 45,000 gills to hhds. Prove, mark, and analyze as in Lesson CXII.

4. Divide the following numbers by 10; by 100; by 1000: 73185; 663587; 741083; 456978.

Divide the same numbers by 11. Divide the same numbers by 12.

5. Write a number of nine places or orders.

Write a number of three full periods.

6. Write a receipt for the payment of $10.50.

7. Divide 1850185 by 56; by 567; by 5678.

CXXI.

1. How do you get one-half of a number? How do you get $\frac{1}{3}$ of a number? $\frac{1}{4}$ of a number? $\frac{1}{5}$ of a number? $\frac{1}{6}$ of a number? Use figures and objects.

2. 2 is $\frac{1}{2}$ of what number? 20 is $\frac{1}{2}$ of what number? 200 is $\frac{1}{2}$ of what number? 300 is $\frac{1}{2}$ of what number?

3. Multiply the following numbers by $5\frac{1}{2}$: 14, 24, 28, 36, 44, 68, 168, 244, 366, 488.

4. $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 12; $\frac{2}{3}$ of 12; $\frac{3}{4}$ of 16; $\frac{2}{3}$ of 15; $\frac{3}{4}$ of 24; $\frac{4}{5}$ of 35; $\frac{5}{6}$ of 36; $\frac{6}{7}$ of 63.

5. 10 is $\frac{1}{10}$ of what number? 15 is $\frac{1}{10}$ of what number? 22 is $\frac{1}{10}$ of what number? 37 is $\frac{1}{10}$ of what number? 48 is $\frac{1}{10}$ of what number?

6. Divide 87568 by 67; by 678; by 6785.

7. Reduce 5 mi., 30 rds., 3 yds., 2 ft., 7 in. to inches.

Mark as in Lesson CXII. and analyze.

8. Change 5 yrs., 200 days, 21 hours, 40 min. to minutes.

Prove and mark as in Lesson CXII., and analyze.

9. How often can a bottle holding 1 qt. and 1 pt. be filled from a hogshead of wine?

How often may the same bottle be filled from 2 gallons?

(Bring the bottle and gallons before the class.)

CXXII.

1. What is a denominate number? What is a compound number? What is reduction? What is reduction descending? What is reduction ascending?

2. Write from memory the lines: "Thirty days hath September," etc.

3. Write all the tables from memory.

4. When was the year 1? When will you be 50 years old?

5. How long since Adam?

6. What year was it 150 years ago?

7. Does it not seem strange that you can measure the height of a steeple with a pole 5 feet long without leaving the ground? Now think on this: Stand the pole on the ground; mark and measure its shadow. Measure the shadow of the steeple. See how many times the *shadow of the stick is contained in the shadow of the steeple.* That many times the pole will give the height of the steeple.

Find the height of your school-house by this plan at recess.

CXXIII.

1. Change these numbers to dollars by putting a point in them: 495 cents; 8755 cents; 1875 cents; 1906 cents; 25025 cents.

2. Change these dollars to cents: $25; $37; $49; $149; $336; $485; $85; $75; $50.

3. Make a bill of purchase of five articles sold in a millinery store. Do not fail to have the town, the date, the name of the merchant, and the name of the purchaser.

4. Divide 3503856 by 7; by 78; by 789.

5. Divide the following numbers by 10, by 100, by 1000: 67532; 87015; 679654; 8792.

Divide the same numbers by 11.

Divide the same numbers by 12.

6. Annex a cipher to $5, remove the dollar mark, and you have dimes; annex another cipher, and you have cents; annex another cipher, and you have mills.

7. Change $25 to dimes; change $25 to cents; change $25 to mills. Change $48 to dimes; change $48 to cents; change $48 to mills.

CXXIV.

1. I had ten hens; each hen raised 15 chickens; the chickens after a time laid 25 eggs each; I sold the eggs at 2 cents apiece; how much money did I get for the eggs? *Ans.* $75.

2. What is the value of 75 horses at $78.75 each?

3. If $7875 will pay for 75 cows, what is one cow worth?

4. If 2 men can do a certain work in 3 days, how many boys can do the work in 1 day if 2 boys can do the work of one man? *Ans.* 12 boys.

5. Make an example involving a pile of wood at so much a cord.

6. Make an example where you imagine the length, width, and height of a room; then ask how many square yards of paper will cover both side walls. Ask how much paper will cover the end walls. Work it, and explain.

CXXV.

1. Multiply the following numbers by $5\frac{1}{2}$: 16, 24, 36, 78, 64.

2. What is a coin? What are the different coins made of?

3. Divide the following numbers by 10, by 100, by 1000: 87658; 73428; 69325; 879543.

Divide the same numbers by 11.

Divide the same numbers by 12.

4. Divide 485098 by 8; by 89; by 897.

5. Write all the tables from memory.

6. Change 5 mi., 80 rods, 3 yds., 2 ft., 10 in. to inches.

CXXVI.

1. Reduce 4 tons, 15 cwt., 25 lbs., 11 oz. to ounces. Prove.

2. Change 3704 gills to higher denomination. Prove.

3. Reduce 1 yr., 280 da., 20 hr., 40 min., 40 seconds to seconds. Prove.

4. Reduce 900 pints to higher denominations. Prove.

5. Change 400 rds. to miles; 400 inches to yards.

6. Bring before the class strings one inch long, one foot long, one yard long, one rod long.

7. Bring paper before the class and cut a square inch, a square foot, a square yard.

8. Have each child bring you a half-dozen or more inch cubes of wood. Show them a two-inch cube, three-inch cube, forms $2\times4\times4$; $3\times4\times6$, etc.

CXXVII.

1. 3 cwt., 25 lbs. = 325 lbs. 4 cwt., 37 lbs. = 437 lbs. 5 cwt., 30 lbs. = ? 16 cwt., 20 lbs. = ? 27 cwt. 27 lbs. = ? 18 cwt., 17 lbs. = ? 36 cwt., 16 lbs. = ? 24 cwt., 4 lbs. = ?

2. What is a denominate number? What is a compound number? Write five of each.

3. Divide 769504 by 9; by 98; by 987.

4. Divide the following numbers by 10, by 100, by 1000: 67348; 281015; 374064; 28925.

Divide the same numbers by 11.

Divide the same numbers by 12.

5. 15 is $\frac{1}{5}$ of what number? 25 is $\frac{1}{7}$ of what number? 18 is $\frac{1}{8}$ of what number? 35 is $\frac{1}{9}$ of what number?

6. $\frac{4}{5}$ of 20; $\frac{4}{5}$ of 30; $\frac{4}{5}$ of 40; $\frac{4}{5}$ of 60; $\frac{5}{6}$ of 60.

7. $\frac{7}{8}$ of 64; $\frac{7}{9}$ of 63; $\frac{5}{9}$ of 54; $\frac{3}{8}$ of 56.

(The teacher will now review from Lesson C., and complete the Fourth Grade work with)

CXXVIII.

Write the tables, and under each table write a table of unit values, as in Lesson XLV., Ex. 4.

FIFTH GRADE EXERCISES.

FIFTH GRADE EXERCISES.

I.

1. Divide 193475 by 23; by 34; by 45; by 56; by 67; by 78.
2. Write from memory the Tables of Dry Measure; of Liquid Measure; of Long Measure.
3. Write from memory the Tables of Time Measure; of Avoirdupois Weight.
4. Reduce 25 bu., 1 pk., 4 qts., 1 pt. to pints. Prove.
5. Change 1801 pts. to higher denominations.
6. Reduce 4 gal., 3 qts., 1 pt. to pints.
7. Reduce 75 pints to higher denominations.
8. Change 5 yds., 2 ft., 7 in. to inches.
9. Reduce 500 inches to higher denominations.

II.

1. What is the difference between reduction descending and reduction ascending?
2. What is a denominate number? Write five denominate numbers.
3. What is a compound number? Write five compound numbers.
4. You can add, subtract, multiply, and divide compound numbers as well as simple numbers.

   Add 4 lbs., 10 oz. and 5 lbs., 12 oz.
5. Take 3 lbs., 12 oz. from 9 lbs., 10 oz.
6. Multiply 3 lbs., 12 oz. by 4.

7. Divide 9 lbs., 12 oz. by 4.

(The teacher should permit and urge every pupil to attempt 4th, 5th, 6th, and 7th examples for himself. Use the proper weights to show these examples objectively to the class.)

8. $\frac{2}{3}$ of 15; $\frac{3}{5}$ of 25; $\frac{5}{6}$ of 42; $\frac{7}{8}$ of 64; $\frac{6}{7}$ of 49; $\frac{5}{9}$ of 72; $\frac{9}{10}$ of 90.

III.

1. Add 5 bu., 3 pks. and 11 bu., 2 pks.

bu.	pks. (4)
5	3
11	2
16	5

bu.	pks. (4)
5	3
11	2
17	1

(The teacher will explain.)

2. Add 5 yds., 2 ft., 9 in. and 6 yds., 1 ft., 11 in.

3. Add 12 lbs., 6 oz.; 15 lbs., 7 oz.; and 18 lbs., 8 oz.

4. Add 5 gal., 1 qt., 1 pt.; 10 gal., 3 qts., 1 pt.; and 3 qts., 1 pt.

5. From 10 hrs., 28 minutes take 8 hrs., 25 minutes.

hrs.	min.
10	28
8	25
2	3

	hrs.	min.
From	10	28
Take	8	39
	1	49

(The teacher will explain.)

6. Write from memory the Tables of Dry Measure, Long Measure, Avoirdupois Weight, Liquid Measure, and Time Measure.

7. From 25 bu., 2 pks. take 18 bu., 3 pks.

8. From 2 bu., 1 pk., 7 qts. take 1 bu., 3 pks., 6 qts.

9. Add 2 yds., 2 ft., 2 in.; 3 yds., 1 ft., 10 in.; 5 yds., 2 ft., 9 in.; 6 yds., 1 ft., 4 in.; and 10 yds., 7 in.

IV.

1. Take 5 gal., 3 qts., 1 pt. five times.

First Method, by Addition.

gal.	qts.	pt.
5	3	1
5	3	1
5	3	1
5	3	1
5	3	1
29	1	1

Second Method, by Multiplication.

gal.	qts.	pt.
5	3	1
		5
25	15	5

Let pupil explain how the two answers may become alike.

2. Divide 57 bu., 3 pks. by 7. Prove.

(Wherever the example calls for proof the teacher should rigidly require it.)

3. Multiply 1 lb., 1 oz., 1 pwt., 1 gr. by 25.

4. Write from memory the Table of Square Measure; of Solid or Cubic Measure.

5. Add 15 sq. yds,, 7 sq. ft., 90 sq. in.; 10 sq. yds., 3 sq. ft., 60 sq. in.; 14 sq. yds., 5 sq. ft., 100 sq. in.

6. From 15 gal., 1 qt., 1 pt. take 10 gal., 3 qts., 1 pt.

7. From 1890 yrs., 6 mo., 10 da. take 1776 yrs., 8 mo., 17 da.

8. If a man travel 26 mi., 150 rds. in 1 day, how far will he travel in 8 days?

9. If a barrel of molasses contains 41 gal., 2 qts., 1 pt., and shall lose by leakage 5 gal., 3 qts., what will remain in the barrel?

V.

1. If a field contains 24 acres, what are $\frac{3}{8}$ of the field worth at $40.50 an acre? *Ans.* $648.

2. Add 5 cu. yds., 16 cu. ft., 400 cu. in.; 16 cu. yds., 13 cu. ft., 1200 cu. in.; 25 cu. yds., 16 cu. ft., 1000 cu. in.

3. Take 5 cu. yds., 16 cu. ft., 1375 cu. in. from 11 cu. yds., 11 cu. ft., 1100 cu. in.

4. If a car carries 25 tons, 10 cwt., 25 lbs. of iron, what do 12 cars carry?

5. Divide 19 bu., 2 pks. by 9. Prove.

6. Divide 10 tons, 3 cwt., 20 lbs. by 8. Prove.

7. Divide 27 yds., 2 ft., 4 in. by 10. Prove.

8. Divide 38 gal., 3 qts., 1 pt., 1 gill by 5. Prove.

9. How many cu. inches in 27 cu. feet?

10. How many cu. inches in 27 cu. yards?

VI.

1. If a barrel of pork is worth 20 dollars, what are $\frac{4}{5}$ of it worth?

2. What is the difference between $\frac{3}{5}$ of 25 and $\frac{5}{8}$ of 24? Use objects before using figures in class.

3. Add 5 hhds., 20 gal., 3 qts., 1 pt.; 25 hhds., 15 gal., 1 qt.; 36 gal., 2 qts., 1 gill; 18 hhds., 3 qts., 3 gills.

4. Multiply 5 wks., 4 da., 20 hrs., 30 min., 30 sec. by 6. Prove.

5. Divide 5 wks., 4 da., 20 hrs., 30 min., 4 sec. by 8. Prove.

6. Take 15 wks., 4 da., 20 hrs., 30 min., 4 sec. from 21 wks., 3 da., 17 hrs., 28 min.

7. I bought at one time 25 hams, weighing each 17 lbs., at $12\frac{1}{2}$ cents a pound; at another time 37 hams, each weighing 16 lbs., at $11\frac{1}{2}$ cents a pound, and at another time 28 hams, each weighing 15 lbs., at 12 cents a pound. What did the whole cost me?

\$171.605.

VII.

1. I bought 2 ton, 5 cwt., 10 lbs. of iron at 5 cents a pound and sold it at a profit of 1 cent a pound; what did I get for it?

2. How many times will a wheel 12 ft. in circumference turn round in going 1 mile? 440 times.

3. Change 5 centuries to days.

4. Add 10 lbs., 10 oz.; 15 cwt., 25 lbs., 6 oz.; 16 cwt., 24 lbs., 12 oz.; 18 cwt., 35 lbs., 9 oz.; 5 tons, 15 cwt., 37 lbs.

5. Take 1 hhd., 37 gal., 2 qts., 1 pt., 1 gill from 3 hhds.

6. Divide 4 hhds., 37 gal., 2 qts., 1 pt., 1 gill by 25.

hhds. gal. qts. pt. gill.
25) 4 37 2 1 1 (0 hhd.
63 gal.
252 gal.
37
25) 289 gal. (11 gal.
25
39
25
14 gal.
4 qts.
56 qts.
2 qts.
25) 58 qts. (2 qts.
50
8 qts.
2 pts.
16 pts.
1 pts.
25) 17 pts. (0 pt.
4 gills.
68
1 gill.
25) 69 gills. (2 gills.
50
19 gills remaining.

7. Multiply 2 hhds., 16 gal., 3 qts., 1 pt. by 25.
8. How many yards in 1 mile?
9. How many pints in 1 hhd.?
10. How many pounds in 1 t.?
11. How many seconds in 1 yr.?

VIII.

1.

mi.	rds.
14	37
28	10
19	16
56	10

From 13 1 4 yds. ft. in.
Take 10 2 11

35 mi. 20 rds.
200

24)43 mi. 280 rds

2.

gal.	qts.	pt.
2	2	1
12	2	0
7	0	0
15	1	1

From 51 1 1 bu. pks. qt.
Take 27 3 0

14 gal. 2 qts. 1 pt.
35

9)34 bu. 3 pks. 4 qts.

3. Find the difference between April 2, 1890, and June 15, 1881. (30 days are considered a month.)

yrs.	mos.	da.
1890	4	2
1881	6	15

yrs.	mos.	da.
1891	5	18
1776	7	4

yrs.	mos.	da.
1891	10	17
1847	11	19

4. Find the exact number of minutes from May 10 to Aug. 10.
5. How many hours in $\frac{1}{2}$ a year?

IX.

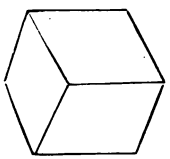
1. Divide 2 yrs., 135 da., 17 hrs. by 72.
2. Multiply 35 cu. yds., 15 cu. ft., 1200 cu. in. by 25.
3. How many cubic inches in 5 cubic yards?
4. What is a square? What is a rectangle?

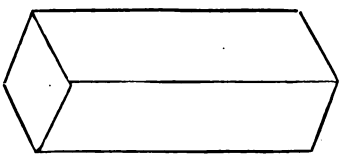
5. Carry to the class one inch, one square inch, one cubic inch; one foot, one square foot, one cubic foot.

6. How long, wide, and deep is a cord of wood?

7. How do you get the area of a surface of a square, or of a rectangle?

8. How do you get the contents of a cube? Of a rectangular solid?





9. What is the difference between a cube and any other rectangular solid? Make one of each out of pasteboard.

10. Divide 455 t., 15 cwt., 24 lbs., 8 oz. by 36.

Write the tables, and under each table construct a table of unit values as in Lesson XLV., Ex. 4.

(As the time of the following lesson should be taken up by the teacher in developing the subject of fractions, Lesson VIII. may be given to the class as seat work.)

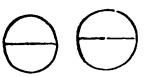
X.

A fraction represents an equal part or equal parts of a unit. Fractions indicate division.

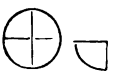
The teacher and pupils should provide a number of disks of pasteboard five inches in diameter. It will facilitate the instruction if half the pupils shall have scissors to divide disks as the teacher divides them.

Let a disk now be divided in two parts, the parts named, and a pupil ordered to place the fraction $\frac{1}{2}$ on the board. The number above the line is called the *numerator* because it tells the *number* of parts taken.

The number below the line is called the *denominator* because it names the *kind* of parts, and shows into how many parts the disk or unit is divided. Cut other disks in halves, and place the fractions on the board, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, $\frac{6}{2}$; at the same time exhibiting the parts of disks which the fractions represent. Let the pupils see that $\frac{2}{2}=1$, $\frac{3}{2}=1\frac{1}{2}$, $\frac{4}{2}=2$, $\frac{5}{2}=2\frac{1}{2}$, after presenting $\frac{3}{2}$ in the form of a disk and a half, $\frac{2}{2}$ in the form of a disk, $\frac{4}{2}$ in the form of two disks, and $\frac{5}{2}$ in the form of two and a dalf disks, thus:

$\frac{2}{2}=1$, $\frac{3}{2}=1\frac{1}{2}$, $\frac{4}{2}=2$ 

Now divide a half and show the fourth on the board, holding up the part represented by the fraction. Show

$\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}=1$, $\frac{5}{4}=1\frac{1}{4}$ 

and so on until your pupils can answer promptly the following questions, showing the parts mentioned, at the same time placing the fractions on the board:

How many halves in a unit?

How many fourths in a unit?

How many fourths in $\frac{1}{2}$?

What are $\frac{2}{2}$ equal to?

What are $\frac{3}{2}$ equal to?

What are $\frac{4}{2}$ equal to?

What are four fourths equal to?

What are five fourths equal to?

What are six fourths equal to?

Now divide the fourths in two parts, exhibiting the parts, and having the fractions $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, etc., written on the board.

$\frac{1}{2}$ is equal to how many eighths?

$\frac{1}{4}$ equals how many eighths?

Divide the disks into three parts, showing the parts, and writing fractions on the board to represent the parts.

Divide the thirds in two parts, and show sixths, at the same time having the fractions to represent the parts.

Divide the thirds into three parts, showing ninths, and placing fractions on the board to represent the parts.

XI.

The teacher can now bring out the distinction of fractions into *proper* and *improper*. Write on the board as below, and as each fraction is written show the parts represented by it, and remark whether it is less or not less than a unit; whether it is properly a part or equal to a unit or more than a unit.

Proper Fractions.	Improper Fractions.
	$\frac{1}{2}$ $\frac{2}{2}$
$\frac{1}{3}$ $\frac{2}{3}$	$\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$
$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$	$\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$
$\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$	$\frac{5}{5}$ $\frac{6}{5}$ $\frac{7}{5}$ $\frac{8}{5}$ $\frac{9}{5}$ $\frac{10}{5}$
$\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$	$\frac{6}{6}$ $\frac{7}{6}$ $\frac{8}{6}$ $\frac{9}{6}$ $\frac{10}{6}$ $\frac{11}{6}$
$\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$	$\frac{7}{7}$ $\frac{8}{7}$ $\frac{9}{7}$ $\frac{10}{7}$ $\frac{11}{7}$ $\frac{12}{7}$ etc.

A proper fraction has a numerator less than its denominator. An improper fraction has a numerator equal to or greater than its denominator.

The teacher will exhibit $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$, $\frac{7}{3}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, $\frac{8}{4}$, $\frac{9}{4}$, $\frac{10}{4}$ $\frac{11}{4}$; in each case arrange the parts to complete whole disks, placing whole numbers and fractions corresponding on the board, thus developing mixed numbers.

A mixed number is a whole number and a fraction in one expression.

The teacher should have cigar-boxes labeled "Halves," "Thirds," "Fourths," etc., so that she may preserve the parts in orderly keeping for future use.

Let the pupil now write fractions and mixed numbers to the number of one hundred, arranged as below:

Proper Fractions.	Improper Fractions.	Mixed Numbers.
$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{7}{8}$	$\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$	$1\frac{1}{2}$ $2\frac{1}{2}$ $2\frac{1}{3}$ $3\frac{1}{4}$
$\frac{9}{10}$ etc.	$\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$ $\frac{8}{3}$ etc.	$4\frac{2}{3}$ $4\frac{3}{4}$ etc.

Require them to write the fractions on the board, as in the diagram in first part of this lesson; and as each fraction is written let some pupil exhibit the parts of disks represented. Let the pupils complete this

TABLE OF PRINCIPLES.

A fraction is .....

Fractions indicate .....

The terms of a fraction .....

The numerator is ..... *and shows* .....

The denominator is ..... *and shows* .....

A proper fraction is .....

An improper fraction is .....

A mixed number is .....

(The teacher should go over the whole ground again as minutely as before, using the disks as freely as possible. It would be well to spend four or five days in Lessons X. and XI. For seat work, besides writing fractions, give some previous lesson. Uncut colored disks should be used with which to make comparisons. Thus, $\frac{1}{3}=\frac{3}{9}$; place one-third on a disk and three-ninths on the third.)

XII.

1. What are the terms of a fraction?

Name the terms in $\frac{2}{4}$; in $\frac{2}{6}$; in $\frac{3}{8}$; in $\frac{2}{8}$; in $\frac{4}{8}$.

$\frac{2}{4}=\frac{1}{2}$; $\frac{2}{6}=\frac{1}{3}$; $\frac{3}{6}=\frac{1}{2}$; $\frac{2}{8}=\frac{1}{4}$; $\frac{4}{8}=\frac{1}{2}$. Illustrate with parts of disks.

Which fraction has lower terms, $\frac{2}{4}$ or $\frac{1}{2}$? $\frac{2}{6}$ or $\frac{1}{3}$? $\frac{3}{6}$ or $\frac{1}{2}$? etc.

If you divide the terms of $\frac{2}{4}$ by 2, you get $\frac{1}{2}$; have you changed the form of the fraction? Yes, from $\frac{2}{4}$ to $\frac{1}{2}$. Have you changed the value? No, because $\frac{2}{4}=\frac{1}{2}$.

2. Which fraction has lower terms, $\frac{2}{6}$ or $\frac{1}{3}$? If you divide the terms of $\frac{2}{6}$ by 2, you get $\frac{1}{3}$. Have you changed the form of the fraction? Yes, from $\frac{2}{6}$ to $\frac{1}{3}$. Have you changed the value of the fraction? No, because $\frac{1}{3}=\frac{2}{6}$. Illustrate with parts of disks.

3. Which fraction has lower terms, $\frac{4}{8}$ or $\frac{1}{2}$? If you divide the terms of $\frac{4}{8}$ by 4, you get $\frac{1}{2}$. Have you changed the form of the fraction? Yes, from $\frac{4}{8}$ to $\frac{1}{2}$. Have you changed the value of the fraction $\frac{4}{8}$? No, because $\frac{4}{8}=\frac{1}{2}$. Illustrate.

Then to change a fraction to its lowest terms, divide both terms by the largest number that will divide them without a remainder.

4. Change these fractions to their lowest terms: $\frac{4}{8}$, $\frac{6}{8}$, $\frac{5}{10}$, $\frac{4}{10}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{10}{12}$, $\frac{6}{12}$.

5. $\frac{10}{12}$, $\frac{10}{15}$, $\frac{12}{15}$, $\frac{12}{18}$, $\frac{16}{18}$, $\frac{15}{20}$, $\frac{12}{20}$, $\frac{18}{24}$.

6. Write from memory the complete

TABLE OF PRINCIPLES.

A fraction is .....

Fractions indicate .....

The terms of a fraction are .....

The numerator is .....*and shows* .....

The denominator is .....*and shows* .....

A proper fraction is .....

An improper fraction is .....

A mixed number is .....

To reduce a fraction to its lowest terms .....

XIII.

1. Let the pupils examine these fractions, and name those that are not equal to a unit, after handling the disks in each case:

$\frac{3}{2}$	$\frac{5}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$
$\frac{4}{3}$	$\frac{6}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{8}{8}$	$\frac{8}{7}$
$\frac{2}{2}$	$\frac{7}{3}$	$\frac{6}{4}$	$\frac{7}{5}$	$\frac{10}{6}$	$\frac{11}{7}$
$\frac{5}{2}$	$\frac{8}{3}$	$\frac{9}{4}$	$\frac{10}{5}$	$\frac{12}{6}$	$\frac{14}{7}$

What kind of fractions are these?

To reduce an improper fraction to a whole or mixed number, divide its numerator by its denominator.

Reduce the above fractions thus: $\frac{3}{2}=1\frac{1}{2}$.

2. Let the pupil examine these mixed numbers, and say whether they can be changed to proper or improper fractions. Change them:

$1\frac{1}{2}$	$4\frac{2}{3}$	$5\frac{1}{3}$	$6\frac{1}{2}$	$7\frac{1}{7}$	$8\frac{1}{7}$
$2\frac{1}{2}$	$4\frac{1}{3}$	$5\frac{2}{3}$	$6\frac{1}{3}$	$7\frac{2}{5}$	$8\frac{2}{3}$
$2\frac{1}{3}$	$4\frac{1}{4}$	$5\frac{1}{4}$	$6\frac{2}{3}$	$7\frac{3}{4}$	$8\frac{3}{4}$
$3\frac{1}{3}$	$4\frac{3}{4}$	$5\frac{3}{4}$	$6\frac{3}{4}$	$7\frac{2}{5}$	$8\frac{4}{5}$

To reduce a mixed number to an improper fraction *multiply the whole number by the denominator of the fraction; to the product add the numerator, and place the sum over the denominator.*

3. What do fractions indicate?

XIV.

1. Write fifteen proper fractions, fifteen improper fractions, and fifteen mixed numbers, and reduce all of them that can be reduced.

2. Would eight men working two weeks at 90 cents a day earn as much money as 15 boys working 3 weeks at 35 cents a day?

3. Write from memory the complete

TABLE OF PRINCIPLES.

(1) *A fraction is* .....
(2) *Fractions indicate* .....
(3) *The terms of* .....
(4) *The numerator is* ..... *and shows* .....
(5) *The denominator is* ..... *and shows* .....
(6) *A proper fraction* .....
(7) *An improper fraction* .....
(8) *A mixed number is* .....
(9) *To reduce a fraction to its lowest terms* .....
(10) *To reduce an improper fraction to* .....
(11) *To reduce a mixed number to* .....

XV.

1. If you multiply both terms of $\frac{1}{2}$ by 2, you get $\frac{2}{4}$. Is the form of the fraction changed? Yes, from $\frac{1}{2}$ to $\frac{2}{4}$. Is the value of the fraction changed? No, because $\frac{2}{4}=\frac{1}{2}$.

2. If you multiply both terms of $\frac{1}{3}$ by 3, you get $\frac{3}{9}$. Is the form of the fraction changed? Yes, from $\frac{1}{3}$ to $\frac{3}{9}$. Is the value of the fraction altered? No, because $\frac{1}{3}=\frac{3}{9}$.

3. Then can you tell how to reduce fractions to higher terms? *Multiply both terms by the same number?*

Change the following fractions to higher terms:

$\frac{1}{2}$	$\frac{8}{9}$	$\frac{5}{9}$	$\frac{3}{7}$	$\frac{1}{9}$
$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{9}$	$\frac{4}{5}$	$\frac{1}{10}$
$\frac{3}{4}$	$\frac{3}{5}$	$\frac{7}{9}$	$\frac{2}{7}$	$\frac{5}{10}$

If you multiply both terms of $\frac{1}{2}$ by 5 you get $\frac{1}{2}=\frac{5}{10}$; both terms of $\frac{2}{3}$ multiplied by 5, gives you $\frac{2}{3}=\frac{10}{15}$.

(Use disks to show equality after multiplying.)

4. How do you reduce fractions to their lowest terms?

Reduce these fractions to their lowest terms:

$\frac{12}{14}$	$\frac{15}{18}$	$\frac{16}{20}$	$\frac{10}{15}$	$\frac{15}{25}$
$\frac{10}{12}$	$\frac{12}{18}$	$\frac{18}{20}$	$\frac{20}{30}$	$\frac{14}{20}$
$\frac{6}{15}$	$\frac{10}{20}$	$\frac{18}{24}$	$\frac{30}{40}$	$\frac{14}{28}$

XVI.

1. How do you reduce improper fractions to whole or mixed numbers?

Reduce these improper fractions:

$\frac{5}{3}$	$\frac{8}{3}$	$\frac{18}{8}$	$\frac{22}{8}$	$\frac{25}{10}$
$\frac{7}{4}$	$\frac{8}{6}$	$\frac{15}{4}$	$\frac{25}{8}$	$\frac{26}{11}$
$\frac{7}{3}$	$\frac{9}{7}$	$\frac{18}{5}$	$\frac{27}{6}$	$\frac{27}{10}$
$\frac{9}{5}$	$\frac{10}{3}$	$\frac{20}{6}$	$\frac{19}{7}$	$\frac{28}{12}$

2. How do you reduce mixed numbers to improper fractions?

Reduce the following whole numbers and mixed numbers to improper fractions:

$5=\frac{10}{2}$	$4\frac{1}{4}=?$	$7\frac{1}{3}=?$	$9=\frac{\ }{7}$	$8\frac{1}{2}=?$
$5\frac{1}{2}=\frac{11}{2}$	$5\frac{3}{4}=?$	$7\frac{2}{3}=?$	$9\frac{1}{2}=?$	$8\frac{2}{3}=?$
$3=\frac{9}{3}$	$5\frac{5}{8}=?$	$7\frac{3}{4}=?$	$9\frac{1}{7}=?$	$8=\frac{\ }{5}$
$3\frac{1}{3}=\frac{10}{3}$	$5\frac{5}{7}=?$	$7\frac{4}{5}=?$	$9\frac{3}{4}=?$	$8\frac{1}{4}=?$
$3\frac{2}{3}=?$	$6\frac{1}{2}=?$	$9\frac{5}{6}=?$	$8\frac{1}{5}=?$	

3. $\frac{3}{4}$ of 8; $\frac{5}{8}$ of 18; $\frac{5}{7}$ of 21; $\frac{3}{5}$ of 20; $\frac{5}{7}$ of 21.

4. Change 10000 pints to hogsheads. Prove.

5. Divide 18763 by 23; by 34; by 45; by 56.

6. I bought from Miller Bros.

235 lbs. sugar at $6\frac{1}{2}$ cents per pound.
345 lbs. salt at $3\frac{1}{2}$ cents per pound.
749 lbs. nails at 5 cents a pound.
837 lbs. butter at 35 cents a pound.

Ans. $357.75.

Write the above in the form of a bill, and show that it was paid. The pupil to be the purchaser.

XVII.

1. Make this pyramid of proper and improper fractions, drawing a line down between the two kinds:

PYRAMID OF FRACTIONS.

Proper. Improper.

$\frac{1}{2}$ | $\frac{2}{2}$

$\frac{1}{3}$ $\frac{2}{3}$ | $\frac{3}{3}$ $\frac{4}{3}$

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ | $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$

$\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ | $\frac{5}{5}$ $\frac{6}{5}$ $\frac{7}{5}$ $\frac{8}{5}$

$\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ | $\frac{6}{6}$ $\frac{7}{6}$ $\frac{8}{6}$ $\frac{9}{6}$ $\frac{10}{6}$

$\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$ | $\frac{7}{7}$ $\frac{8}{7}$ $\frac{9}{7}$ $\frac{10}{7}$ $\frac{11}{7}$ $\frac{12}{7}$

2. Arrange these fractions in four columns, placing mixed numbers in the first column, improper fractions in the second column, proper fractions to be reduced to lowest terms in third column, and proper fractions to be reduced to higher terms in fourth column. Show what each fraction is equal to.

$4\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{8}$, $\frac{8}{5}$, $\frac{9}{10}$, $\frac{10}{12}$, $5\frac{1}{8}$, $9\frac{3}{4}$, $4\frac{2}{3}$, $\frac{5}{8}$, $\frac{15}{25}$, $\frac{25}{35}$, $\frac{27}{36}$, $\frac{4}{7}$, $7\frac{1}{2}$, $\frac{5}{11}$, $6\frac{4}{7}$, $\frac{30}{42}$, $5\frac{1}{8}$, $8\frac{2}{5}$, $\frac{13}{15}$, $\frac{45}{50}$, $\frac{11}{7}$, $\frac{24}{36}$, $\frac{10}{13}$, $5\frac{5}{7}$, $\frac{13}{4}$, $\frac{19}{8}$, $\frac{25}{7}$, $\frac{38}{8}$, $10\frac{2}{3}$, $\frac{22}{10}$, $\frac{18}{30}$, $\frac{35}{8}$, $\frac{2}{3}$, $\frac{65}{9}$.

3. From \$30 take $37\frac{1}{2}$ cents; from \$19.10 take \$7.79; 25 cents less $12\frac{1}{2}$ cents; from \$25 take 25 cents; take 49 cents from \$49; \$187 less \$.187.

4. Write from memory with the utmost neatness and care the complete Table of Principles, adding the following for the 12th principle: *To reduce fractions to higher terms multiply both terms by the same number.*

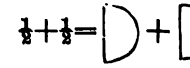
(The teacher should place on the wall a chart of prime numbers from 1 to 97, explain to her pupils the subject of factoring, and that these numbers cannot be factored.)

XVIII.

1. $\frac{1}{2}+\frac{1}{2}$=what? $\frac{1}{3}+\frac{2}{3}$=what? $\frac{1}{4}+\frac{3}{4}$=what? $\frac{2}{4}+\frac{3}{4}$=?

$\frac{3}{5}+\frac{4}{5}$=?	$\frac{5}{6}+\frac{3}{6}$=?	$\frac{7}{8}+\frac{1}{8}$=?
$\frac{5}{8}+\frac{4}{8}$=?	$\frac{3}{8}+\frac{7}{8}$=?	$\frac{5}{3}+\frac{4}{3}$=?
$\frac{3}{5}+\frac{4}{5}$=?	$\frac{5}{9}+\frac{7}{9}$=?	$\frac{7}{5}+\frac{3}{5}$=?
$\frac{5}{7}+\frac{4}{7}$=?	$\frac{4}{7}+\frac{3}{7}$=?	$\frac{4}{7}+\frac{3}{7}$=?

(Let the pupil determine by his attainment what these fractions equal, placing the whole on the board in view of the class. Then show them with the disks what the fractions are equal to.)

$\frac{1}{2}+\frac{1}{2}=$  $\frac{1}{3}+\frac{2}{3}=$

(This is extremely important, and teachers are urged not to fail to illustrate every one of the above examples with parts of disks.)

(Do likewise with the following.)

2.

$\frac{3}{4}-\frac{1}{4}$=?	$\frac{6}{10}-\frac{5}{10}$=?	$\frac{9}{10}-\frac{1}{10}$=?
$\frac{5}{6}-\frac{3}{6}$=?	$\frac{9}{10}-\frac{7}{10}$=?	$\frac{7}{9}-\frac{2}{9}$=?
$\frac{7}{8}-\frac{5}{8}$=?	$\frac{8}{10}-\frac{5}{10}$=?	$\frac{8}{9}-\frac{1}{9}$=?
$\frac{8}{9}-\frac{4}{9}$=?	$\frac{5}{6}-\frac{3}{6}$=?	$\frac{2}{3}-\frac{1}{3}$=?
$\frac{4}{3}-\frac{2}{3}$=?	$1-\frac{1}{9}$=?	

Work the above in view of the class.

3. If $15 will buy a barrel of sugar, what would $\frac{2}{3}$ of a barrel cost?

4. If a ton of hay is worth $39, what is $\frac{1}{3}$ of a ton worth?

5. If one quarter of a beef is worth $2.50, what is a whole beef worth?

6. Divide 230875 by 56; by 67; by 78; by 89.

(The teacher should make a chart containing composite numbers from 4 to 150, show how they differ from prime numbers. Spend ten minutes a week factoring them.)

XIX.

1. $\frac{10}{12}$, $\frac{15}{20}$, $\frac{25}{30}$, $\frac{36}{45}$, $\frac{27}{45}$, $\frac{18}{60}$, $\frac{13}{26}$, $\frac{28}{42}$, $\frac{35}{63}$. Write over these fractions how they must be reduced, and then reduce them.

2. $5\frac{1}{2}$, $4\frac{1}{8}$, $9\frac{1}{7}$, $6\frac{1}{6}$, $8\frac{1}{7}$, $5\frac{5}{7}$, $10\frac{2}{7}$, $11\frac{5}{8}$, $9\frac{7}{3}$. Write over these numbers how they must be reduced; then reduce them.

3. $\frac{2}{3}$, $\frac{5}{3}$, $\frac{9}{5}$, $\frac{11}{7}$, $\frac{17}{8}$, $\frac{22}{9}$, $\frac{35}{8}$, $\frac{45}{7}$, $\frac{59}{8}$, $\frac{68}{9}$. Write over these fractions how they must be reduced; then reduce them.

4. Write over these fractions how they must be reduced to higher terms, then reduce them. $\frac{1}{3}$, $\frac{2}{7}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{7}{9}$, $\frac{5}{6}$, $\frac{5}{7}$, $\frac{4}{7}$, $\frac{9}{10}$.

5. What is the twelfth principle? See Lesson XVII., Ex. 4.

How do you reduce fractions to their lowest terms?

6. How do you reduce fractions to higher terms?

7. If you multiply the numerator of a fraction alone, do you increase or diminish the number of parts?

8. Write as neatly and beautifully as you can the complete Table of Principles. Do not omit the twelfth principle.

XX.

1. 4 is $\frac{1}{3}$ of what number? 4 is $\frac{2}{3}$ of what number? 8 is $\frac{1}{4}$ of what number? 8 is $\frac{2}{4}$ of what number?

2. 9 is $\frac{1}{4}$ of what number? 9 is $\frac{3}{4}$ of what number? 10 is $\frac{2}{5}$ of what number? 12 is $\frac{3}{5}$ of what number?

(To explain these examples properly, stretch a string or wire in easy view of the class. On the string or wire should be placed one hundred or more ends of spools or large wooden buttons or beads. For the first example, draw apart from the

rest of the objects 12 objects, separate them in three groups of 4 each. Covering two of the groups, ask: "4 is $\frac{1}{3}$ of what number?" Illustrate the other examples in the same way.)

3. A grocer bought 25 bushels of apples at \$.85 a bushel, and sold them for 25 cents a peck; what was his profit? *Ans.* \$3.75.

4. A certain school has 144 pupils; $\frac{3}{8}$ of them are in the fourth grade, $\frac{2}{9}$ are in the fifth grade, the remainder are in the sixth grade; how many in each grade?

5. Write 25 proper fractions, 25 improper fractions, 25 mixed numbers.

6. Write the pyramid of fractions as in Lesson XVII.

XXI.

1. Write these fractions in three columns, placing proper fractions in one column, improper fractions in one column and mixed numbers in another column. Show what each fraction is equal to. $\frac{15}{30}$, $\frac{15}{6}$, $8\frac{1}{7}$, $\frac{30}{65}$, $\frac{27}{8}$, $9\frac{5}{8}$, $\frac{48}{64}$, $\frac{35}{8}$, $8\frac{3}{4}$, $\frac{56}{63}$, $\frac{47}{9}$, $12\frac{5}{7}$, $\frac{36}{72}$, $\frac{49}{8}$, $13\frac{5}{7}$. Write under each column, telling how each column must be reduced.

2. Raise these fractions to higher terms: $\frac{5}{8}$, $\frac{7}{9}$, $\frac{10}{15}$, $\frac{9}{13}$, $\frac{8}{15}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{7}{10}$, $\frac{8}{11}$.

3. Add these fractions:

$\frac{1}{2}+\frac{3}{2}+\frac{5}{2}=\frac{9}{2}$ $\quad \frac{5}{7}+\frac{4}{7}+\frac{3}{7}=?$ $\quad \frac{5}{12}+\frac{4}{12}+\frac{6}{12}=?$

$\frac{3}{4}+\frac{2}{4}+\frac{8}{4}=1\frac{3}{4}$ $\quad \frac{3}{8}+\frac{5}{8}+\frac{7}{8}=?$ $\quad \frac{8}{11}+\frac{4}{11}+\frac{5}{11}=?$

(Illustrate with parts of disks.)

4. Find the difference

$\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$ $\quad \frac{10}{11}-\frac{1}{11}=?$ $\quad \frac{25}{30}-\frac{4}{30}=?$

$\frac{16}{32}-\frac{4}{32}=?$ $\quad \frac{10}{14}-\frac{5}{14}=?$ $\quad \frac{7}{10}-\frac{3}{10}=?$

$\frac{17}{30}-\frac{5}{30}=?$ $\quad \frac{18}{40}-\frac{15}{40}=?$ $\quad \frac{7}{9}-\frac{2}{9}=?$

$\frac{15}{16}-\frac{10}{16}=?$ $\quad \frac{24}{36}-\frac{15}{36}=?$ $\quad \frac{17}{35}-\frac{16}{35}=?$

5. Reduce 34323 ounces to higher denominations. Prove.

6. I bought 12 bu. and 3 pks. of apples at $.80 a bushel. What did they cost me? What was the cost of 1 pk.

XXII.

1. How many gallons in 20 qts? How many pints in half a gallon?

2. At 84 cents a gallon what will 3 qts. of molasses cost?

3. Which cost the more, 240 lbs. of sugar at 6 cts. a pound or 30 gallons of molasses at 10 cts. a quart?

4. What part of a day is 1 hour? 6 hours? 8 hours?

5. How many minutes in January?

6. How many ounces in $2\frac{1}{2}$ lbs.?

7. What part of a pound is 1 ounce? 6 oz.? 8 oz.? 10 oz.?

(Bring the weights before the class.)

8. What is $\frac{1}{4}$ of a ton of iron worth at 3 cents a pound?

9. How many yards in 1 mile? how many feet?

10. How many inches in $\frac{1}{2}$ yard? How many yards in $\frac{9}{2}$ yards?

11. How many rods in 1 mile? What part of a mile is 1 rd.? What part of a mile is 40 rds.?

12. How many sq. ft. in a board 2 ft. wide and 10 ft. long?

13. How many square yards in a hall 6 ft. wide and 30 ft. long?

14. How many square feet in a board 10 inches wide and 10 feet long?

XXIII.

1. How many cents in 40 dollars? How many dimes in 40 dollars? How many eagles in 500 dimes?

2. Change the following numbers to cents: $8.75, $9.37, $15.12, $14.01, $8.59, $9.63, $25.34.

3. Change the following numbers to dollars: 455 cents, 875 cents, 937 cents, 646 cents, 1875 cents.

4. Divide 338705 by 89; by 98.

5. At $3\frac{1}{2}$ dollars a week, what will a boy earn in 2 weeks? What will he earn in 6 weeks?

6. If a man walk 4 miles an hour, how far will he walk in $8\frac{1}{2}$ hours?

7. A boy had an orange, and gave his sister $\frac{1}{4}$ of it; how much had he left? If he had given her $\frac{2}{5}$ of the orange, how much would he have had left?

(Use real orange.)

8. How long, wide, and deep is a cord of wood?

9. How many cords in a pile of wood 32 feet long, 10 feet wide, and 8 feet high, and what is it worth at $3.75 a cord? *Ans.* $75.

XXIV.

1. If a boy earns $\$\frac{3}{8}$ a day, what will he earn in 2 days?

Ans. He will earn 2 times $\$\frac{3}{8}$ or $\frac{3}{8}\times 2=\$\frac{6}{8}$. $\$\frac{6}{8}=\$\frac{3}{4}$.

2. If a yard of ribbon is worth $\$\frac{3}{4}$, how much are 3 yards worth?

3. If 1 pound of sugar is worth $\$\frac{1}{8}$, what are 3 pounds worth? what are 5 lbs. worth? what are 12 lbs. worth?

4. If $1 will buy $\frac{3}{7}$ of a yard of cloth, how much will $9 buy? How much will $12 buy?

(Carry goods and dollars to class.)

5. If a horse eat $\frac{3}{8}$ of a bushel of oats in one day, how much will he eat in 8 days? in 10 days?

6. Multiplying the numerator multiplies the number of parts. $\frac{2}{3}\times 4=\frac{8}{3}$. Multiply the denominator of $\frac{3}{4}$ by 4, and you get $\frac{3}{16}$, and $\frac{3}{16}$ is less than $\frac{3}{4}$. If you divide the denominator, you make the parts larger. Divide the denominator of $\frac{3}{4}$ by 2, and you get $\frac{3}{2}$, which is greater in value than $\frac{3}{4}$.

7. Then this will be your thirteenth principle: *Multiplying a fraction consists in multiplying the numerator or dividing the denominator.*

8. Write the complete Table of Principles. Do not omit the thirteenth principle.

XXV.

1. $\frac{3}{8}\times 2=\frac{6}{8}=\frac{3}{4}$ $\quad \frac{5}{12}\times 2=?$ $\quad \frac{3}{9}\times 7=?$
$\frac{4}{5}\times 3=\frac{12}{5}=2\frac{2}{5}$ $\quad \frac{2}{3}\times 11=?$ $\quad \frac{3}{11}\times 3=?$
$\frac{8}{13}\times 6=?$ $\quad \frac{3}{4}\times 15=?$ $\quad \frac{5}{7}\times 6=?$
$\frac{5}{14}\times 5=?$ $\quad \frac{9}{15}\times 9=?$ $\quad \frac{4}{5}\times 16=?$
$\frac{7}{8}\times 8=?$ $\quad \frac{7}{10}\times 6=?$ $\quad \frac{14}{15}\times 3=?$
$\frac{5}{6}\times 12=?$ $\quad \frac{5}{9}\times 10=?$ $\quad \frac{8}{11}\times 7=?$
$\frac{7}{9}\times 7=?$ $\quad \frac{5}{18}\times 12=?$

2. $\frac{2}{2}-\frac{1}{2}=?$ $\quad \frac{5}{6}-\frac{4}{6}=?$ $\quad \frac{15}{16}-\frac{4}{16}=?$ $\quad \frac{16}{22}-\frac{11}{22}=?$
$\frac{2}{3}-\frac{1}{3}=?$ $\quad \frac{6}{7}-\frac{5}{7}=?$ $\quad \frac{16}{18}-\frac{5}{16}=?$ $\quad \frac{17}{22}-\frac{5}{22}=?$
$\frac{7}{9}-\frac{6}{9}=?$ $\quad \frac{8}{9}-\frac{5}{9}=?$

3. Fourteenth principle: *Dividing the numerator divides the fraction; dividing the denominator increases the value of the fraction.*

(Let the pupils try to show this at the board.)

XXVI.

1. $\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=1$ $\quad \frac{6}{7}+\frac{2}{7}+\frac{1}{7}=?$ $\quad \frac{5}{12}+\frac{4}{12}+\frac{3}{12}=?$
2. $\frac{2}{3}+\frac{4}{3}=\frac{6}{3}=2$ $\quad \frac{3}{8}+\frac{4}{8}+\frac{5}{8}=?$ $\quad \frac{3}{15}+\frac{9}{15}+\frac{11}{15}=?$

ations.

$.80 a
as the

y pints

olasses

t 6 cts.
quart?
urs? 8

8 oz.?

cents a

et?
y yards

t of a

l 10

e

3. $\frac{3}{4}+\frac{3}{4}=?$ $\frac{5}{9}+\frac{7}{9}+\frac{4}{9}=?$ $\frac{9}{18}+\frac{7}{18}+\frac{10}{18}=?$

4. $\frac{4}{5}+\frac{4}{5}=?$ $\frac{3}{9}+\frac{5}{9}+\frac{3}{9}=?$ $\frac{5}{20}+\frac{8}{20}+\frac{12}{20}=?$

5. $\frac{6}{8}+\frac{6}{8}=?$ $\frac{1}{10}+\frac{5}{10}+\frac{7}{10}=?$ $\frac{3}{24}+\frac{5}{24}+\frac{7}{24}=?$

(Use the disks, each pupil preparing the parts for one example.)

6. Reduce these fractions to higher terms by multiplying both terms of each fraction by 6 or by some other number: $\frac{2\times6}{3\times6}=\frac{12}{18}$, $\frac{5}{9}$, $\frac{10}{12}$, $\frac{8}{10}$, $\frac{13}{14}$.

7. $\frac{3\times6}{5\times6}=\frac{18}{30}$, $\frac{8}{10}$, $\frac{5}{7}$, $\frac{7}{12}$, $\frac{14}{15}$.

8. $\frac{5}{8}$, $\frac{7}{12}$, $\frac{6}{11}$, $\frac{9}{11}$, $\frac{15}{18}$.

9. $\frac{7}{8}$, $\frac{9}{10}$, $\frac{4}{9}$, $\frac{5}{10}$, $\frac{8}{9}$.

XXVII.

1. Reduce these fractions to their lowest terms by dividing both terms by the largest divisor: $\frac{4}{8}$, $\frac{10}{12}$, $\frac{10}{18}$, $\frac{15}{20}$, $\frac{16}{24}$, $\frac{24}{30}$, $\frac{30}{40}$.

2. $\frac{5}{10}$, $\frac{14}{18}$, $\frac{10}{20}$, $\frac{15}{25}$, $\frac{16}{20}$, $\frac{24}{36}$, $\frac{30}{50}$.

3. $\frac{3}{9}$, $\frac{14}{21}$, $\frac{10}{25}$, $\frac{15}{30}$, $\frac{16}{28}$, $\frac{24}{40}$, $\frac{30}{60}$.

4. $\frac{2}{4}$, $\frac{14}{28}$, $\frac{10}{30}$, $\frac{15}{35}$, $\frac{16}{30}$, $\frac{24}{42}$, $\frac{30}{70}$.

How do you reduce a fraction to its lowest terms?

How do you reduce a fraction to higher terms?

5. Reduce these mixed numbers to improper fractions: $2\frac{1}{2}=\frac{5}{2}$, $5\frac{1}{5}$, $7\frac{1}{8}$, $9\frac{1}{4}$, $12\frac{2}{3}$, $18\frac{5}{9}$.

6. $3\frac{1}{3}=\frac{10}{3}$, $5\frac{4}{5}$, $7\frac{3}{8}$, $9\frac{3}{8}$, $14\frac{3}{4}$, $18\frac{3}{12}$.

7. $3\frac{2}{3}$, $6\frac{2}{3}$, $8\frac{3}{5}$, $10\frac{4}{8}$, $14\frac{4}{5}$, $25\frac{5}{8}$.

8. $4\frac{2}{3}$, $6\frac{5}{8}$, $8\frac{6}{7}$, $10\frac{5}{7}$, $15\frac{3}{7}$, $25\frac{4}{7}$.

9. Write the prime numbers to 97.

XXVIII.

1. Reduce these improper fractions to whole or mixed numbers: $\frac{10}{3}$, $\frac{14}{7}$, $\frac{18}{8}$, $\frac{25}{8}$, $\frac{50}{11}$, $\frac{65}{9}$.

2. $\frac{11}{4}$, $\frac{16}{8}$, $\frac{20}{4}$, $\frac{35}{8}$, $\frac{50}{9}$, $\frac{65}{7}$.

3. $\frac{12}{5}$, $\frac{16}{4}$, $\frac{20}{5}$, $\frac{45}{8}$, $\frac{50}{10}$, $\frac{65}{8}$.

4. $\frac{13}{8}$, $\frac{15}{8}$, $\frac{20}{8}$, $\frac{49}{8}$, $\frac{50}{8}$, $\frac{65}{8}$.

5. Work the following examples by division:

$\frac{4}{2} \div \frac{2}{2} = 2$ $\frac{7}{5} \div \frac{3}{5} = ?$ $\frac{12}{5} \div \frac{3}{5} = ?$ $\frac{11}{4} \div \frac{3}{4} = ?$

6. $\frac{6}{3} \div \frac{2}{3} = 3$ $\frac{8}{3} \div \frac{2}{3} = ?$ $\frac{15}{6} \div \frac{4}{6} = ?$ $\frac{22}{6} \div \frac{5}{6} = ?$

7. $\frac{8}{4} \div \frac{3}{4} = 2\frac{2}{3}$ $\frac{9}{3} \div \frac{3}{3} = ?$ $\frac{18}{7} \div \frac{3}{7} = ?$ $\frac{35}{8} \div \frac{4}{8} = ?$

8. $\frac{9}{4} \div \frac{5}{4} = ?$ $\frac{10}{4} \div \frac{2}{4} = ?$ $\frac{20}{8} \div \frac{3}{8} = ?$ $\frac{40}{7} \div \frac{4}{7} = ?$

(Illustrate with parts of disks.)

(Examples in Lessons XXV., XXVI., XXVII., and XXVIII. are intended not only for seat work, but should be worked orally in class at least four times during Fifth Grade year.)

XXIX.

1. $\frac{4}{5}$ of 20; $\frac{5}{6}$ of 30; $\frac{4}{7}$ of 35; $\frac{3}{8}$ of 24; $\frac{6}{7}$ of 21.

2. 8 is $\frac{2}{3}$ of what number? 15 is $\frac{3}{5}$ of what number? 20 is $\frac{5}{6}$ of what number? 35 is $\frac{5}{7}$ of what number?

MODEL. $\frac{2}{3}$ of some number is 8.
$\frac{1}{3}$ of that number is $\frac{1}{2}$ of 8 or 4.
$\frac{3}{3}$, or the number, are 3 times 4, or 12.
Therefore 8 is $\frac{2}{3}$ of 12.

PROOF. $\frac{1}{3}$ of 12 is 4.
$\frac{2}{3}$ of 12 are 8.

3. What are the terms of a fraction?

4. How do you raise fractions to higher terms?

5. Which is greater, $\frac{1}{2}$ or $\frac{4}{8}$? $\frac{1}{3}$ or $\frac{2}{6}$? $\frac{1}{3}$ or $\frac{5}{15}$? $\frac{2}{5}$ or $\frac{4}{10}$? $\frac{4}{9}$ or $\frac{8}{18}$? $\frac{3}{4}$ or $\frac{9}{12}$? $\frac{4}{6}$ or $\frac{8}{12}$? $\frac{3}{8}$ or $\frac{6}{16}$?

6. Raise $\frac{1}{2}$ to sixths. $\frac{1}{2} = \frac{3}{6}$. Multiply both terms of $\frac{1}{2}$ by 3, and you get $\frac{3}{6}$.

Change $\frac{1}{4}$ to eighths; $\frac{2}{3}$ to ninths; $\frac{3}{4}$ to twelfths; $\frac{4}{5}$ to tenths; $\frac{5}{8}$ to sixteenths; $\frac{5}{7}$ to fourteenths; $\frac{6}{8}$ to sixteenths; $\frac{3}{5}$ to fifteenths; $\frac{9}{12}$ to twenty-fourths; $\frac{5}{9}$ to eighteenths.

7. Reduce $\frac{3}{7}$ to fourteenths; $\frac{7}{8}$ to twenty-fourths; $\frac{5}{6}$ to thirtieths; $\frac{3}{4}$ to twentieths; $\frac{4}{7}$ to twenty-eighths; $\frac{5}{9}$ to thirty-sixths.

3. $\frac{1}{4}+\frac{2}{4}=?$ $\frac{5}{9}+\frac{7}{9}+\frac{4}{9}=?$ $\frac{9}{18}+\frac{7}{18}+\frac{10}{18}=$

4. $\frac{4}{5}+\frac{6}{5}=?$ $\frac{3}{9}+\frac{5}{9}+\frac{2}{9}=?$ $\frac{5}{20}+\frac{8}{20}+\frac{12}{20}=?$

5. $\frac{6}{8}+\frac{6}{8}=?$ $\frac{1}{10}+\frac{5}{10}+\frac{7}{10}=?$ $\frac{3}{24}+\frac{5}{24}+\frac{7}{24}=?$

(Use the disks, each pupil preparing the *parts* for one ample.)

6. Reduce these fractions to higher *terms by* multiplying both terms of each fraction by 6 or by so other number: $\frac{2\times6}{3\times6}=\frac{12}{18}$, $\frac{5}{9}$, $\frac{10}{12}$, $\frac{8}{10}$, $\frac{13}{14}$.

7. $\frac{3\times6}{5\times6}=\frac{18}{30}$, $\frac{8}{10}$, $\frac{5}{7}$, $\frac{7}{12}$, $\frac{14}{15}$.

8. $\frac{5}{8}$, $\frac{7}{12}$, $\frac{6}{11}$, $\frac{9}{11}$, $\frac{15}{18}$.

9. $\frac{7}{8}$, $\frac{9}{10}$, $\frac{4}{9}$, $\frac{5}{10}$, $\frac{8}{9}$.

XXVII.

1. Reduce these fractions to their lowest terms by dividing both terms by the largest divisor: $\frac{4}{8}$, $\frac{10}{12}$, $\frac{10}{15}$, $\frac{15}{20}$, $\frac{16}{24}$, $\frac{24}{30}$, $\frac{30}{40}$.

2. $\frac{5}{10}$, $\frac{14}{18}$, $\frac{10}{20}$, $\frac{15}{25}$, $\frac{16}{20}$, $\frac{24}{36}$, $\frac{30}{50}$.

3. $\frac{3}{9}$, $\frac{14}{21}$, $\frac{10}{25}$, $\frac{15}{30}$, $\frac{16}{28}$, $\frac{24}{40}$, $\frac{30}{60}$.

4. $\frac{2}{4}$, $\frac{14}{28}$, $\frac{10}{30}$, $\frac{15}{35}$, $\frac{16}{30}$, $\frac{24}{42}$, $\frac{30}{70}$.

How do you reduce a fraction to its lowest terms?

How do you reduce a fraction to higher terms?

5. Reduce these mixed numbers to improper fractions: $2\frac{1}{2}=\frac{5}{2}$, $5\frac{1}{5}$, $7\frac{1}{8}$, $9\frac{1}{4}$, $12\frac{2}{3}$, $18\frac{5}{9}$.

6. $3\frac{1}{3}=\frac{10}{3}$, $5\frac{4}{5}$, $7\frac{3}{8}$, $9\frac{3}{8}$, $14\frac{3}{4}$, $18\frac{3}{12}$.

7. $3\frac{2}{3}$, $6\frac{2}{3}$, $8\frac{3}{5}$, $10\frac{4}{8}$, $14\frac{4}{5}$, $25\frac{5}{8}$.

8. $4\frac{2}{3}$, $6\frac{5}{8}$, $8\frac{6}{7}$, $10\frac{5}{7}$, $15\frac{3}{7}$, $25\frac{4}{7}$.

9. Write the prime numbers to 97.

XXVIII.

1. Reduce these improper fractions to whole or mixed numbers: $\frac{10}{3}$, $\frac{14}{7}$, $\frac{18}{8}$, $\frac{25}{8}$, $\frac{50}{11}$, $\frac{65}{9}$.

2. $\frac{11}{4}$, $\frac{16}{8}$, $\frac{20}{4}$, $\frac{35}{8}$, $\frac{50}{9}$, $\frac{65}{7}$.

3. $\frac{12}{5}$, $\frac{16}{4}$, $\frac{20}{5}$, $\frac{45}{8}$, $\frac{50}{10}$, $\frac{65}{8}$.

4. $\frac{18}{6}$, $\frac{15}{3}$, $\frac{20}{6}$, $\frac{49}{8}$, $\frac{50}{8}$, $\frac{65}{6}$.

$+\frac{5}{9}$.

…t be
…mon
…elect
…r that

…enom-
…omina-

…nths; 6
…venths;

… are $\frac{10}{2}$.

$\frac{3}{4}=\frac{20}{}$
$\frac{4}{5}=\frac{20}{}$
$\frac{3}{10}=\frac{20}{}$
$\frac{7}{10}=\frac{20}{}$

… $\frac{8}{9}$ of 72.

…er? 24 is $\frac{4}{9}$ of what num-

XXX.

1. Write as neatly as you can the complete Table of Fourteen Principles.

2. Write five proper fractions, five improper fractions, and five mixed numbers.

3. What does the numerator of a fraction show?

4. What does the denominator of a fraction show?

5. How many thirds in 1? in 2? in 3?

6. How many fourths in 1? in 2? in 3?

7. How many fifths in 1? in 2? in 3?

8. How many sixths in 1? in 2? in 3?

9. How many sevenths in 1? in 2? in 3?

10. One is what part of 2? 1 is what part of 3?

11. One is what part of 4? 1 is what part of 5?

12. What is a fraction?

13. What do you understand by $\frac{1}{2}$ of any thing? by $\frac{1}{3}$ of any thing? by $\frac{2}{3}$ of any thing? by $\frac{3}{4}$ of any thing? by $\frac{4}{5}$ of any thing?

XXXI.

1. Raise $\frac{1}{2}$ to fourteenths; $\frac{1}{4}$ to sixteenths; $\frac{1}{5}$ to twentieths; $\frac{3}{4}$ to sixteenths; $\frac{2}{5}$ to twentieths; $\frac{3}{8}$ to sixteenths; $\frac{4}{5}$ to twentieths; $\frac{1}{2}$ to sixteenths.

2. To raise a fraction to a given denominator you must multiply both terms by the same number, and if you look closely at both denominators you can easily tell what your multiplier is: $\frac{3}{8}=\frac{9}{24}$? Do you not see that 3 is the multiplier? then $\frac{3}{8}=\frac{9}{24}$. $\frac{5}{7}=\frac{\ }{14}$? $\frac{3}{10}=\frac{\ }{30}$? $\frac{3}{12}=\frac{\ }{36}$? $\frac{3}{9}=\frac{\ }{18}$? $\frac{5}{6}=\frac{\ }{30}$? $\frac{7}{11}=\frac{\ }{22}$? $\frac{5}{8}=\frac{\ }{24}$? $\frac{5}{6}=\frac{\ }{48}$? $\frac{7}{8}=\frac{\ }{40}$? $\frac{8}{10}=\frac{\ }{40}$? $\frac{4}{5}=\frac{\ }{35}$?

3. $\frac{3}{4}$ and $\frac{2}{4}$ are $\frac{5}{4}$. You can add fractions only when they have the same denominator, or what some children call a *common denominator.*

Have $\frac{1}{3}$ and $\frac{2}{3}$ a common denominator?

Ans. Yes, 3.

Have $\frac{1}{3}$ and $\frac{1}{4}$ a common denominator?

Ans. No.

To add $\frac{1}{3}$ and $\frac{1}{4}$ you must change them to other fractions having a like denominator.

$\frac{1}{3}=\frac{4}{12}$ $\quad \frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$

$\frac{1}{4}=\frac{3}{12}$

4. What do fractions indicate?

XXXII.

1. Add $\frac{1}{5}+\frac{1}{2}$; $\frac{1}{3}+\frac{1}{5}$; $\frac{1}{6}+\frac{1}{3}$; $\frac{1}{4}+\frac{1}{5}$; $\frac{2}{3}+\frac{3}{4}$; $\frac{3}{4}+\frac{4}{5}$; $\frac{5}{8}+\frac{5}{9}$.

2. Add $\frac{5}{7}+\frac{4}{8}$; $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$; $\frac{1}{3}+\frac{1}{5}+\frac{1}{6}$.

Remember the fractions you add must first be changed to other fractions having a *common denominator*. And remember too, to select some number for a common denominator that the denominators will divide.

Thus: $\frac{1}{5}=\frac{2}{10}$. Take 10 for a common denominator, because the denominators 5 and 2 will divide it.
$\frac{1}{2}=\frac{5}{10}$.

3. Change 5 to halves; 8 to thirds; 4 to tenths; 6 to fifths; 9 to fourths; 7 to eighths; 10 to sevenths; 12 to fifths.

MODEL. In 1 there are $\frac{2}{2}$, in 5 there are $\frac{10}{2}$.

4.

$\frac{3}{4}=\frac{}{12}$	$\frac{1}{5}=\frac{}{15}$	$\frac{1}{2}=\frac{}{20}$	$\frac{3}{4}=\frac{}{20}$
$\frac{5}{6}=\frac{}{12}$	$\frac{1}{3}=\frac{}{15}$	$\frac{1}{4}=\frac{}{20}$	$\frac{4}{5}=\frac{}{20}$
$\frac{2}{3}=\frac{}{12}$	$\frac{2}{5}=\frac{}{15}$	$\frac{1}{5}=\frac{}{20}$	$\frac{3}{10}=\frac{}{20}$
$\frac{1}{2}=\frac{}{12}$	$\frac{2}{3}=\frac{}{15}$	$\frac{1}{10}=\frac{}{20}$	$\frac{7}{10}=\frac{}{20}$

5. $\frac{4}{5}$ of 60; $\frac{3}{8}$ of 48; $\frac{6}{7}$ of 42; $\frac{8}{9}$ of 72.

6. 12 is $\frac{3}{4}$ of what number? 24 is $\frac{4}{9}$ of what number?

$\frac{3}{4}$ of some number is 12.
$\frac{1}{4}$ of the number is $\frac{1}{3}$ of 12, or 4.
$\frac{4}{4}$, or the number, are 4 times 4=16.
Therefore 12 is $\frac{3}{4}$ of 16.
PROOF. $\frac{1}{4}$ of 16=4
$\frac{3}{4}$ of 16=3×4=12.

7. What do fractions indicate?

XXXIII.

1. How many sheets of paper in 35 reams?
2. How many yards in 1 mile? how many feet?
3. How many pounds make 1 ton?
4. How many pints in 1 hogshead? how many gills?
5. How many seconds in a year of 365 days?
6. How many square inches in 1 square yard?
7. How many cubic inches in 1 cubic yard?
8. What is the value of 5 kegs of nails at $4\frac{1}{2}$ cents a pound?
9. What is the value of 5 barrels of flour at $3\frac{1}{2}$ cents a pound?
10. How many units in 5 gross?
11. How many dozen in 5 gross?
12. What is the value of 8 bushels of almonds at $5\frac{1}{2}$ cents a pint?
13. How many grains in 25 Troy pounds?

(The numerator of a fraction is a dividend; the denominator, a divisor.)

XXXIV.

1.

wks.	da.	hrs.	min.	sec.
35	4	15	35	35
17	3	10	20	33
18	6	22	44	44
33	1	17	23	22

	bu.	pks.	qts.	pts.
From	4	2	5	1
Take	1	3	6	1

2. Multiply 31 gal., 3 qts., 1 pt., 3 gills by 35.

3. Divide 1328 lbs., 2 oz., 13 pwt., 12 gr. by 37.

4. Divide 273045 by 234; by 345; by 456.

5. If $11.25 will pay a carpenter for 3 day's work, how many days will $33.75 pay for?

6. How many cubic feet of earth will be taken in digging a cellar 54 ft. long, 20 ft. wide, 10 ft. deep? How many cubic yards?

7. What does *dividend* mean?

8. What is a *denominate number?*

9. What is a compound number?

XXXV.

1. Find the sum, difference, product, and quotient of 98 and 7448, and be careful to mark every term.

2. $\frac{3}{4}$ of $24; $\frac{5}{8}$ of $64; $\frac{4}{5}$ of $40; $\frac{6}{7}$ of $63.

3. $15 is $\frac{3}{8}$ of what number? $30 is $\frac{3}{10}$ of what number?

4. Reduce $\frac{15}{8}$, $\frac{17}{9}$, $\frac{35}{8}$, $\frac{63}{12}$ to mixed numbers. Write how you do it.

5. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{3}{7}$, $\frac{7}{9}$ to higher terms. Write how you do it.

6. Reduce $\frac{3}{6}$, $\frac{8}{18}$, $\frac{15}{24}$, $\frac{20}{28}$, $\frac{21}{35}$ to their lowest terms. Write how you do it.

7. Reduce $3\frac{1}{4}$, $4\frac{2}{5}$, $12\frac{7}{8}$, $23\frac{6}{7}$ to improper fractions. Write how you do it.

(Use the disks.)

8. Reduce 4 to halves; 5 to thirds; 7 to fifths.

9. $\frac{2}{3}=\frac{}{18}$? $\frac{3}{4}=\frac{}{36}$? $\frac{5}{7}=\frac{}{35}$? $\frac{4}{9}=\frac{}{45}$?

10. Do $\frac{1}{2}$ and $\frac{1}{5}$ equal $\frac{2}{7}$? *Ans.* No, because they cannot be added until they have the same denominator.

$\frac{1}{2}+\frac{1}{5}=$? $\frac{2}{3}+\frac{3}{4}=$? $\frac{4}{5}+\frac{5}{8}=$? $\frac{2}{7}+\frac{1}{2}+\frac{1}{14}=$?

XXXVI.

1. $\frac{7}{8}-\frac{2}{3}$. If you take $\frac{2}{3}$ from $\frac{7}{8}$ will you have $\frac{5}{5}$?
Ans. No, because you cannot subtract unless the fractions have the same denominator.
$\frac{7}{8}-\frac{2}{3}=\frac{}{24}-\frac{}{24}=?$
2. $\frac{2}{3}-\frac{1}{8}=?$ $\frac{4}{9}-\frac{1}{3}=?$ $\frac{3}{8}-\frac{1}{5}=?$ $\frac{11}{15}-\frac{3}{5}=?$
3. $\frac{1}{3}\times 2=\frac{2}{3}$. $\frac{2}{3}\times 4=\frac{8}{3}$. $\frac{3}{5}\times 4=?$ $\frac{5}{8}\times 5=?$
4. Multiply $\frac{2}{3}$ by $\frac{3}{5}$; $\frac{5}{8}\times\frac{5}{6}$; $\frac{3}{7}\times\frac{2}{3}$.
Multiply the numerators for a new numerator and multiply the denominators for a new denominator.
5. $\frac{5}{7}\times\frac{3}{8}=?$ $\frac{2}{9}\times\frac{3}{10}=?$ $\frac{1}{2}\times\frac{3}{5}\times\frac{2}{7}=?$ $\frac{3}{4}\times\frac{5}{6}\times\frac{8}{10}=?$
6. $\frac{7}{8}\div\frac{3}{8}=?$ $\frac{9}{10}\div\frac{2}{10}=?$ $\frac{14}{16}\div\frac{3}{16}=?$ $\frac{16}{18}\div\frac{4}{18}=?$
7. $\frac{7}{8}\div\frac{2}{3}$. You cannot divide $\frac{7}{8}$ by $\frac{2}{3}$ unless they have the same denominator. Change both fractions to other fractions having 24 for their denominator. $\frac{7}{8}=\frac{21}{24}$ and $\frac{2}{3}=\frac{16}{24}$. Now you have $\frac{7}{8}\div\frac{2}{3}=\frac{21}{24}\div\frac{16}{24}=\frac{21}{16}$, $\frac{21}{16}=1\frac{5}{16}$.
8. $\frac{3}{4}\div\frac{3}{8}=?$ $\frac{7}{10}\div\frac{3}{4}=?$ $\frac{8}{9}\div\frac{2}{3}=?$

XXXVII.

1. What does the numerator of a fraction show.
What does the denominator of a fraction show?
2. Which is greater, $\frac{1}{3}$ or $\frac{1}{4}$? $\frac{2}{3}$ or $\frac{3}{4}$? $\frac{5}{9}$ or $\frac{3}{7}$? $\frac{8}{10}$ or $\frac{9}{10}$? $\frac{8}{10}$ or $\frac{4}{5}$? $\frac{4}{5}$ or $\frac{5}{6}$? $\frac{1}{3}$ or $\frac{4}{9}$?
You need not try to tell which is greater until you have changed the fractions to other fractions having the same denominator. $\frac{1}{3}=\frac{4}{12}$, $\frac{1}{4}=\frac{3}{12}$. Now you can see that $\frac{1}{3}$ is greater than $\frac{1}{4}$.
3. $\frac{2}{3}\times\frac{5}{8}=?$ $\frac{5}{6}\times 2\frac{1}{2}=?$ $3\frac{1}{3}\times\frac{3}{4}=?$ $\frac{4}{5}\times 8=?$ $\frac{1}{2}\times\frac{3}{4}\times\frac{5}{6}=?$ $4\frac{1}{4}\times 4\frac{1}{4}=?$ $5\frac{1}{2}\times 5\frac{1}{2}=?$
4. $\frac{3}{4}\div\frac{2}{3}=?$ $\frac{9}{10}\div\frac{3}{5}=?$ $\frac{7}{8}\div\frac{5}{8}=?$ $\frac{4}{7}\div\frac{5}{9}=?$ $2\frac{1}{2}\div\frac{1}{3}=?$ $4\frac{1}{3}\div\frac{5}{8}=?$ $5\frac{1}{6}\div\frac{1}{2}=?$ $4\frac{3}{7}\div\frac{4}{5}=?$
5. Write the complete Table of Principles.
Do not omit twelfth, thirteenth, and fourteenth principles.

XXXVIII.

1. Copy this table of proper and improper fractions.

Proper.							Improper.					
					$\frac{1}{2}$		$\frac{2}{2}$					
				$\frac{1}{3}$	$\frac{2}{3}$		$\frac{3}{3}$	$\frac{4}{3}$				
			$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$		$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$			
		$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$		$\frac{5}{5}$	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$		
	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$		$\frac{6}{6}$	$\frac{7}{6}$	$\frac{8}{6}$	$\frac{9}{6}$	$\frac{10}{6}$	
$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$		$\frac{7}{7}$	$\frac{8}{7}$	$\frac{9}{7}$	$\frac{10}{7}$	$\frac{11}{7}$	$\frac{12}{7}$ etc.

2. Reduce $\frac{8}{3}$, $\frac{15}{4}$, $\frac{21}{6}$, $\frac{42}{15}$ to mixed numbers.
3. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$ to fractions having higher terms.
   Reduction of fractions is a process of changing their form without altering their value.
4. Reduce $\frac{15}{24}$, $\frac{18}{30}$, $\frac{24}{36}$, $\frac{35}{45}$ to their lowest terms.
   When is a fraction at its lowest terms?
5. Change $8\frac{2}{3}$, $15\frac{4}{5}$, $26\frac{6}{7}$, $35\frac{5}{18}$ to improper fractions.
6. Add $\frac{5}{8}+\frac{7}{8}+\frac{10}{8}+\frac{13}{8}$. Add $\frac{1}{3}+\frac{5}{6}+\frac{7}{12}$.
7. $\frac{17}{9}-\frac{14}{9}=?$ $\frac{17}{8}-\frac{3}{5}=?$ $\frac{12}{7}-\frac{7}{14}=?$
8. $\frac{2}{3}\times\frac{5}{2}=?$ $\frac{3}{8}\times\frac{3}{4}\times\frac{1}{6}=?$ $\frac{1}{8}\times\frac{5}{7}\times\frac{16}{1}=?$
9. $\frac{15}{8}\div\frac{3}{8}=?$ $\frac{18}{6}\div\frac{3}{6}=?$ $\frac{27}{4}\div\frac{9}{4}=?$
10. $\frac{11}{2}\div\frac{3}{4}=?$ You have learned that you can divide one fraction by another by changing them to the same denominator, and by dividing one numerator by the other; but we will now give you a shorter rule, and you must remember it.

    To divide a fraction, *invert the divisor, and work as in multiplication.*

$\frac{7}{8}\div\frac{3}{4}=\frac{7}{8}\times\frac{4}{3}=\frac{28}{24}=1\frac{4}{24}=1\frac{1}{6}$.

XXXIX.

1. Write how to reduce a fraction to its lowest terms. Make your own example and explain.

2. Write how to reduce an improper fraction to a whole or mixed number. Make your own example and explain.

3. Write how to reduce mixed numbers to improper fractions. Make an example and explain.

4. Write how to add fractions. Make your example and explain.

5. Write how to subtract fractions. Make your example and explain.

6. Write how you multiply fractions. Make your example and explain.

7. Write how to divide a fraction by a fraction. Explain.

8. How many halves in 1? in 2? in 3? in 4?

9. How many thirds in 1? in 2? in 3? in 4?

10. How many fourths in 1? in 3? in 5? in 7?

11. How many fifths in 1? in 2? in 6? in 8?

XL.

1. How many sixths in 1? in 2? in 5? in 9?

2. How many sevenths in 1? in 3? in 5? in 10?

3. How many eighths in 1? in 3? in 6? in 11?

4. Change 4 to halves; 5 to fifths; 6 to thirds.

5. Change 7 to fourths; 9 to eighths; 10 to tenths.

6. $\frac{4}{5}$ of 50; $\frac{5}{6}$ of 72; $\frac{7}{12}$ of 96; $\frac{11}{12}$ of 36.

7. $\frac{2}{3}=\frac{\quad}{30}$; $\frac{5}{6}=\frac{\quad}{42}$; $\frac{5}{8}=\frac{\quad}{88}$; $\frac{8}{9}=\frac{\quad}{81}$.

8. 15 is $\frac{3}{5}$ of what number? 16 is $\frac{4}{9}$ of what number?

9. 36 is $\frac{3}{12}$ of what number? 25 is $\frac{5}{8}$ of what number?

10. Divide 371049 by 456; by 567; by 678.

11. Add \$25.75, 37½ cents, 5 dollars 62½ cents, \$48.09, 22½ cents, 805 dollars, 65 dollars, 87½ cents.

XLI.

1.

wks.	da.	hrs.	min.	sec.
17	5	22	40	45
35	3	18	27	27
18	1	15	15	18
24	6	22	25	25

	gal.	qts.	pts.	gi.
From	63	2	1	2
Take	49	3	1	3

ton.	cwt.	lbs.	oz.
10	15	15	12

2. Multiply 10 ton. 15 cwt. 15 lbs. 12 oz. by 8.

3. Divide 2234 yds., 2 ft., 10 in. by 49.

4. Divide 2234 yds., 2 ft., 3 in by 9.

5. How long would a string be that would reach round your room 5 times?

6. How many cubic inches in a stone step that is 7 ft. long, 3 ft. wide, and 6 in. thick?

Can you multiply feet by inches?

7. How many square inches in the surface of your teacher's table?

8. Divide 205096 by 789; by 879; by 987.

XLII.

1. Work and read each of these examples by both of the models following them.

$\frac{2}{3} \div \frac{3}{4} = ?$ $\quad \frac{5}{6} \div \frac{4}{5} = ?$ $\quad \frac{9}{10} \div \frac{2}{5} = ?$ $\quad \frac{15}{10} \div \frac{1}{2} = ?$

$12 \div \frac{5}{6} = ?$ $\quad 2\frac{1}{2} \div 6 = ?$ $\quad \frac{3}{8} \div \frac{3}{4} = ?$ $\quad \frac{5}{9} \div 1\frac{1}{3} = ?$

$\frac{16}{5} \div \frac{7}{8} = ?$ $\quad \frac{3}{10} \div 1\frac{1}{2} = ?$ $\quad \frac{5}{11} \div \frac{1}{2} = ?$ $\quad 6\frac{1}{2} \div \frac{3}{4} = ?$

FIRST MODEL.

$\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$.

Analysis.

$\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$.

$\frac{8}{12}$ divided by $\frac{9}{12}$ equals $\frac{8}{9}$.

By first model, reduce both fractions to a common denominator before dividing.

SECOND MODEL.

$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.

Analysis.

$\frac{2}{3}$ divided by $\frac{3}{4}$ equals $\frac{2}{3}$ multiplied by $\frac{4}{3}$, which equals $\frac{8}{9}$.

By second method, invert the divisor and proceed as in multiplication.

XLIII.

(The teacher will give some former lesson for seat work for this recitation, to be handed in on paper, and will devote the time of the recitation to developing the fact that *there are 231 cubic inches in a gallon.*

Have a box made 11 inches long, 7 inches wide, and 3 inches deep, all in the clear. Paste heavy writing paper over all the edges, thus making the box so secure that it will hold water. Bring before the class a gallon measure full of water, which pour into the box. Then, emptying the box, fill it with inch cubes, which shall have been provided.

Now make practical application of the information developed.

Bring before the class a water-tight box, have the pupils take its dimensions in inches, find the solid contents of the box in cubic inches, and divide by 231 inches, to learn how many gallons in the box.

Then verify your work, objectively, by filling the box with water with your gallon measure.)

XLIV.

1. How do you get the area or surface of a wall or floor or field?

2. How many square feet in each wall of your room?

3. How many acres in a field 200 rods long and 80 rods wide?

4. How do you get the solid or cubic contents of a block of wood or marble, or of a cellar, or of a pile of wood?

5. How many cords in a pile of wood 40 ft. long, 32 ft. wide, and 10 ft. high?

6. How many gallons in a box 11 in. long, 7 in. wide, and 3 in. deep?

(Let the pupils make boxes with these dimensions.)

7. How many gallons will fill a box 30 in. long, 11 in. wide, and 7 in. deep?

8. How many gallons can be contained in a box 11 ft. long, 7 ft. wide, and 3 ft. deep? *Ans.* 1728 gal.

Can you divide feet by inches?

(Let the pupils draw figures to show the difference between one foot, one square foot, and one cubic foot.)

XLV.

1. Reduce 15375 pints to bushels. Prove.
2. Reduce 364857 seconds to higher denominations. Prove.
3. Change 40 mi. and 3 yds. to inches. Prove.
4. How many square yards in 1 square mile?
5. How many square yards in a lot 180 ft. long and 100 ft. wide?
6. If a car runs 20 miles an hour, how far will it run in $10\frac{1}{2}$ hours?
7. If a car runs $10\frac{1}{2}$ miles an hour, how long will it take to run 147 miles? *Ans.* 14 hr.
8. What is the difference between 1 square inch and 2 inches square? between 3 square inches and 3 inches square? between 6 square inches and 7 inches square?

(The teacher should require the pupils to bring scissors and paper, and work these examples objectively. Make free use of scissors and paper.)

XLVI.

1. 1 is what part of 4? *Ans.* 1 is $\frac{1}{4}$ of 4.
   2 is what part of 4? *Ans.* 2 is $\frac{2}{4}$ or $\frac{1}{2}$ of 4.
   3 is what part of 4?

(The teacher should place 4, or 5, or 6, or more objects on the table in sight of the class. Let some child raise one object and tell what part of the group of objects it is. Then raise two objects, three objects, and so on, telling each time the fraction. Some child should at the same time write on the board, "1 is $\frac{1}{4}$ of 4," "2 is $\frac{2}{4}$ of 4," as the case may be.)

2. 1 is what part of 7? 2 is what part of 7? 5 is what part of 7? 4 is what part of 8? 5 is what part of 8? 3 is what part of 10?

3. 5 is what part of 12? 7 is what part of 13? 9 is what part of 14? 8 is what part of 15?

4. 3 times $\frac{2}{3}$. 4 times $\frac{3}{4}$. 5 times $\frac{3}{5}$.
3 times $\frac{2}{3}=\frac{6}{3}=2$. 4 times $\frac{3}{4}=\frac{12}{4}=3$.
6 times $\frac{1}{2}$=? 7 times $\frac{2}{3}$=? 8 times $\frac{1}{3}$=?
5 times $\frac{1}{2}$=? 8 times $\frac{4}{5}$=? 9 times $\frac{5}{7}$=?
7 times $\frac{3}{10}$=? 10 times $\frac{1}{3}$=?

5. 9 times $\frac{3}{8}$=? 7 times $\frac{5}{6}$=? 8 times $\frac{4}{7}$=?
7 times $\frac{5}{9}$=?

6. 3 times $2\frac{1}{3}$=?
Ans. 3 times $2\frac{1}{3}=6\frac{3}{3}$, $\frac{3}{3}=1$, $6+1=7$; therefore 3 times $2\frac{1}{3}$ equals 7.

7. 4 times $4\frac{1}{2}$=? 4 times $2\frac{1}{2}$=? 5 times $3\frac{1}{2}$=?
6 times $4\frac{2}{3}$=? 7 times $3\frac{1}{3}$=? 8 times $1\frac{1}{3}$=?
8 times $4\frac{1}{2}$=? 5 times $2\frac{2}{3}$=? 9 times $4\frac{1}{5}$=?

XLVII.

1. $\frac{5}{8}\div\frac{1}{2}$=? $2\frac{1}{2}\div\frac{3}{4}$=? $4\frac{2}{3}\div\frac{5}{8}$=?
$5\frac{1}{2}\div\frac{2}{3}$=? $3\frac{3}{4}\div 4$=? $\frac{5}{6}\div\frac{2}{3}$=?
$\frac{9}{10}\div\frac{1}{3}$=? $\frac{1}{2}\div\frac{1}{16}$=? $\frac{2}{3}\div\frac{3}{8}$=?

First Method. Second Method.
$\frac{5}{8}\div\frac{1}{2}=\frac{5}{8}\div\frac{4}{8}=\frac{5}{4}=1\frac{1}{4}$. $\frac{5}{8}\div\frac{1}{2}=\frac{5}{8}\times\frac{2}{1}=\frac{10}{8}=1\frac{1}{4}$.

Work all the examples by both methods.

2. Reduce $\frac{15}{24}$, $\frac{25}{30}$, $\frac{48}{84}$ to their lowest terms.

3. Reduce $15\frac{5}{9}$, $25\frac{5}{8}$, $35\frac{3}{5}$ to improper fractions.

4. Reduce $\frac{5}{8}$, $\frac{7}{8}$, $\frac{8}{9}$ each to higher terms.

5. Change $\frac{65}{13}$, $\frac{57}{14}$, $\frac{96}{25}$ to mixed numbers.

XLVIII.

1. Change $\frac{2}{3}$ to ninths; $\frac{5}{8}$ to forty-eighths; $\frac{7}{9}$ to sixty-thirds; $\frac{5}{7}$ to forty-ninths; $\frac{3}{5}$ to twentieths; $\frac{7}{8}$ to fifty-sixths.

2. $\frac{5}{12}=\frac{?}{24}$ $\frac{5}{12}=\frac{?}{36}$ $\frac{5}{12}=\frac{?}{48}$
$\frac{5}{8}=\frac{?}{40}$ $\frac{5}{8}=\frac{?}{64}$ $\frac{5}{8}=\frac{?}{72}$
$\frac{3}{10}=\frac{?}{50}$ $\frac{3}{10}=\frac{?}{70}$ $\frac{3}{10}=\frac{?}{90}$

3. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$. $\frac{3}{4}+\frac{5}{6}+\frac{7}{12}=?$ $\frac{5}{9}+\frac{2}{3}+\frac{5}{6}=?$

4. Add $\frac{1}{3}$, $\frac{5}{8}$, $\frac{7}{15}$, $\frac{1}{4}$.

You may not be able to tell what the common denominator should be in this example. When that is the case, find by the following plan a number that your denominators will divide: Place the denominators in a line. *Divide by any prime number that will divide two or more of the numbers. Divide quotients and undivided numbers in the same way, and so on until you cannot divide two of the quotients. Multiply all the divisors and last quotients, and their product will be suitable for your common denominator.* This number is called the least common multiple of the denominators.

3)	3	8	15	4
2)	1	8	5	4
2)	1	4	5	2
		2	5	1

$3\times2\times2\times5\times2=120$

XLIX.

1. Find the least common multiple of these denominators, and use it for a common denominator:

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{4}{15}$.

Thus

2)	2	3	8	12	15
3)	1	3	4	6	15
2)	1	1	4	2	5
			2	1	5

$2\times3\times2\times2\times5=120$.

$\frac{1}{2}=\frac{?}{120}$ $\frac{2}{3}=\frac{?}{120}$ $\frac{5}{8}=\frac{?}{120}$ $\frac{7}{12}=\frac{?}{120}$ $\frac{4}{15}=\frac{?}{120}$

2. Find the least common multiple of 20, 45, 60; 50, 36, 72; 24, 64, 84; 18, 36, 80.

3. $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}=\frac{120}{360}=\frac{1}{3}$.

There is a way to shorten this work, called cancellation. *Cancellation is a method of canceling or rejecting a factor common to the numerator and denominator.* Thus $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}=\frac{2}{6}=\frac{1}{3}$.

(The teacher will show the class how $\frac{1}{3}$ is obtained by cancellation.)

4. Shorten the following examples by cancellation:

$\frac{5}{8}\times\frac{8}{16}\times\frac{16}{10}=?$ $\quad$ $\frac{1}{2}\times\frac{5}{8}\times\frac{8}{7}\times\frac{2}{5}=?$

$\frac{7}{10}\times\frac{2}{5}\times\frac{5}{7}=?$ $\quad$ $\frac{3}{8}\times\frac{24}{36}\times\frac{12}{16}=?$

5. Divide 4305769 by 2345; by 3456; by 4567.

L.

1. Write from memory the rule to find the least common multiple of several numbers. Select four numbers and find their least common multiple.

2. Write the definition of *cancellation.* Make an example and use *cancellation* to shorten the work, but remember you can use cancellation only when you are multiplying fractions or when you have numbers arranged as in the relation of dividend and divisor.

3. *A factor of a number is a maker of a number, of course a divisor.* You are often required to use *prime factors.* You may know what numbers are prime numbers by remembering that they cannot be divided, except by themselves or by 1. The following prime numbers are those you will use most frequently: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc. Complete this list of all the prime numbers to 97.

4. Find the sum, difference, product, and quotient of $\frac{3}{4}$ and $\frac{1}{2}$. Thus:

$\frac{3}{4}=\frac{3}{4}$ $\quad$ Min. $\frac{3}{4}=\frac{3}{4}$

$\frac{1}{2}=\frac{2}{4}$ $\quad$ Sub. $\frac{1}{2}=\frac{2}{4}$

Sum $\frac{5}{4}=1\frac{1}{4}$ $\quad$ Dif. $\frac{1}{4}$

Multipli-cand. Multi-plier. Prod-uct.

$\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$.

Divi-dend. Divi-sor. Quo-tient.

$\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\times\frac{2}{1}=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$.

LI.

1. 5 times $\frac{3}{4}$. 9 times $\frac{5}{8}$. 8 times $\frac{4}{5}$.
2. 5 times $2\frac{1}{2}$. 9 times $3\frac{5}{8}$. 8 times $4\frac{4}{5}$.
3. Reduce $\frac{7}{8}\frac{2}{0}$ and $\frac{6}{8}\frac{5}{5}$ to their lowest terms.
4. Change $\frac{1}{1}\frac{3}{8}\frac{5}{}$ and $\frac{4}{2}\frac{3}{5}\frac{5}{}$ to mixed numbers.
5. Change $75\frac{5}{18}$ and $94\frac{3}{5}$ to improper fractions.
6. Reduce $\frac{1}{1}\frac{3}{8}$ and $\frac{1}{1}\frac{3}{7}$ to higher terms.
7. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{8}$, $3\frac{3}{4}$.
8. Find difference between $\frac{5}{8}$ and $\frac{3}{4}$.
9. Multiply $\frac{2}{3}\times 4\frac{1}{2}$; $4\frac{1}{2}\times\frac{3}{8}$.
10. By both methods, $2\frac{1}{3}\div\frac{2}{3}$.

LII.

1. Find the sum, difference, product, and quotient of $\frac{9}{10}$ and $\frac{3}{5}$.

(The pupil should mark all the terms as in Lesson L., Ex. 4.)

2. 5 is what part of 14? 9 is what part of 17?
3. 42 is $\frac{3}{4}$ of what number? 56 is $\frac{4}{5}$ of what number?
4. $\frac{2}{3}$ of 360 = ? $\frac{3}{4}$ of 480 = ? $\frac{5}{8}$ of 540 = ?
5. Write what a prime number is.
   Write the list of prime numbers up to 97.
6. Find least common multiple of 20, 64, 96.
7. Work this by cancellation

$$\frac{2\times 4\times 5\times 14\times 27}{10\times 8\times 28\times 18} =$$

8. One ham weighs $16\frac{3}{4}$ lbs., another $15\frac{1}{2}$ lbs. What do both weigh? What is their value at $12\frac{1}{2}$ cents a pound? *Ans.* $403\frac{1}{8}$ cts., or $\$4.03\frac{1}{8}$.
9. How many cubic inches in 1 gallon?

LIII.

1. How many rods of fence will inclose a field two miles square? Draw the field.

2. In how many days will a horse eat 50 bushels of oats, if he shall eat 8 quarts twice a day?

3.

m.	rds.	yds.	ft.	in.
5	130	2	2	10
2	110	2	1	9
8	140	2	2	8

	m.	rds.	yds.	ft.	in.
From	5	130	1	1	7
Take	3	140	2	2	8

4. Multiply 8 m. 140 rds. 2 yds. 2 ft. 8 in. by 16.

5. Divide 86 wks. 4 da. 20 hrs. 42 min. 40 sec. by 8.

6. Divide 693 wks., 3 da., 21 hrs., 41 min., 20 sec., by 64.

7. Reduce 40576 ounces to higher denominations.

LIV.

1. Write the rules for dividing fractions by both methods.

Work these examples by both methods:

$\frac{9}{10} \div \frac{1}{3} = ?$ $2\frac{1}{2} \div \frac{5}{8} = ?$ $3\frac{1}{4} \div \frac{3}{4} = ?$

2. Write when a fraction is at its lowest terms.

Reduce $\frac{50}{80}$, $\frac{36}{48}$, $\frac{64}{84}$ to their lowest terms.

3. Reduce $\frac{145}{18}$, $\frac{164}{22}$, $\frac{175}{9}$.

4. Reduce $15\frac{3}{8}$, $17\frac{4}{5}$, $14\frac{2}{3}$.

5. Find the least common multiple of 36, 84, 120.

6. Write a list of the prime numbers up to 97.

7. To find the factors of 60, divide by prime numbers. $2\times2\times3\times5$ are the factors of 60, because when multiplied they *make* 60.

$$\begin{array}{r} 2)60 \\ 2)30 \\ 3)15 \\ \hline 5 \end{array}$$

In the same way find the factors of 120, 180, 144, 250, 248.

8. Divide 303033 by 5678; by 6789.

LV.

1. 3 times $\frac{1}{2}$ = ? 5 times $\frac{2}{3}$ = ? 8 times $\frac{3}{4}$ = ?
   6 times $\frac{3}{8}$ = ? 8 times $\frac{3}{8}$ = ? 9 times $\frac{5}{6}$ = ?
   10 times $\frac{5}{7}$ = ? 10 times $\frac{5}{9}$ = ? 10 times $\frac{7}{10}$ = ?
2. 8 times $2\frac{1}{2}$ = ? 9 times $3\frac{1}{3}$ = ? 11 times $3\frac{1}{4}$ = ?
   10 times $4\frac{1}{8}$ = ? 7 times $3\frac{3}{5}$ = ? 8 times $4\frac{4}{5}$ = ?
3. $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$ = ? $2\frac{1}{3}\times\frac{3}{7}\times\frac{1}{5}$ = ? $4\frac{1}{5}\times\frac{4}{7}\times\frac{2}{3}$ = ?
   $8\frac{1}{6}\times\frac{5}{7}\times\frac{12}{5}$ = ? $3\frac{4}{5}\times\frac{2}{38}\times 2\frac{1}{2}$ = ? $\frac{1}{8}\times\frac{12}{13}\times\frac{26}{5}$ = ?

(These are examples in which to use cancellation.)

Write what cancellation is.

4. $\frac{3}{4}=\frac{}{24}$? $\frac{4}{5}=\frac{}{20}$? $\frac{5}{6}=\frac{}{18}$? $\frac{5}{9}=\frac{}{63}$?
   $\frac{5}{8}=\frac{}{120}$? $\frac{4}{9}=\frac{}{72}$? $\frac{2}{5}=\frac{}{35}$? $\frac{3}{7}=\frac{}{35}$?

5. Change 2 to thirds; 4 to fifths; 6 to eighths; 10 to tenths; 8 to sevenths.

6. Write a receipt for the payment of $90.90.

7. 1 is what part of 2? 1 is what part of 8? 3 is what part of 9? 7 is what part of 15?

8. What do fractions indicate? Prove with disks.

9. Measure the length, width, and depth of a box, and tell how many gallons it will hold.

LVI.

1. Make a bill of purchases of five articles from a hardware store. Work it.

2. 36 is $\frac{9}{10}$ of what number? 48 is $\frac{3}{16}$ of what number?

3. $\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$ = ? $\frac{1}{3}+\frac{2}{3}+\frac{5}{3}$ = ? $\frac{1}{8}+\frac{3}{8}+\frac{9}{8}$ = ?
4. $2\frac{1}{2}+3\frac{1}{2}$ = ? $5\frac{1}{3}+4\frac{1}{2}$ = ? $6\frac{3}{5}+2\frac{1}{7}$ = ?

   Thus: $2\frac{1}{2}$ $5\frac{1}{3}=5\frac{2}{6}$ $6\frac{3}{5}=6\frac{2}{3}\frac{1}{5}$

   $3\frac{1}{2}$ $4\frac{1}{2}=4\frac{3}{6}$ $2\frac{1}{7}=2\frac{5}{35}$

   $5\frac{2}{2}=6$ $9\frac{5}{6}$ $8\frac{26}{35}$

5. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$ = ? $\frac{1}{8}+\frac{3}{4}+\frac{1}{2}$ = ? $\frac{5}{9}+\frac{5}{18}+\frac{5}{3}$ = ?

6. $2\frac{2}{3}+3\frac{3}{4}+5\frac{5}{6}=?$ $1\frac{1}{8}+5\frac{3}{4}+6\frac{1}{2}=?$ $1\frac{5}{9}+6\frac{5}{16}+2\frac{3}{5}=?$

7. $\frac{7}{8}-\frac{2}{3}=?$ $\frac{5}{6}-\frac{1}{4}=?$ $12\frac{1}{2}-3\frac{1}{4}=?$ $6\frac{2}{7}-5\frac{1}{8}=?$

8. How many fourths in $\frac{1}{2}$? How many ninths in $\frac{1}{3}$?

9. How many tenths in $\frac{2}{5}$? How many twelfths in $\frac{3}{6}$?

LVII.

The pupil will notice how much alike these words are:

DIVISOR.	MULTIPLE.
DIVISION.	MULTIPLI—CATION.

1. What are the *divisors* and *multiples* of 12?

Divisors.		Multiples.
2, 3, 4, 6.	12.	1 time 12=12.
		2 times 12=24.
		3 times 12=36.
		4 times 12=48.
		5 times 12=60, etc.

2. How many divisors has 12? *Ans.* Four.

3. How many multiples has 12?

Ans. Thousands of them.

(The teacher should extend the list of multiples until the pupils realize that the number is unlimited.)

Every smart boy or girl will see that a multiple is a *product.*

4. Name the divisors of 15.

Ans. The divisors are 3 and 5.

5. Name the multiples of 15. 1 time 15=15.
2 times 15=30.
3 times 15=45.
4 times 15=60, etc.

6. What divisor is common to 12 and 15?

Ans. 3.

7. What multiple is common to 12 and 15?

Ans. 60.

(The teacher should give practice after the above plan every three or four days, placing the numbers on the board thus:

Divisors.		Multiples.
2, 4, 8	16	16, 32, 48, 64, 80, 96, etc.
2, 4, 5, 10	20	20, 40, 60, 80, etc.

Have children write divisors on the left, and on the right have twenty multiples or more. Mark the common divisors and common multiples in heavier chalk, to distinguish them. Then call for the *greatest common divisor* and *least common multiple.*)

LVIII.

1. Write the complete Table of Principles.

2. If you multiply the numerator of a fraction, what effect does it have on the fraction? If you multiply the denominator?

3. If you divide the numerator of a fraction, what effect does it have on the fraction? If you divide the denominator?

4. Suppose you divide both terms by the same number, what effect does it have?

5. Suppose you multiply both terms, what effect does it have?

(Let the pupil place a fraction on the board and illustrate the above questions by operating on the terms as asked.)

6. How do you add and subtract fractions that have not the same denominator? *Ans. Find the least common multiple of the denominators, and use that for a common denominator.*

LIX.

1. Find the sum, difference, product, and quotient of $2\frac{1}{2}$ and $1\frac{1}{4}$, and mark all the terms.

2. 5 is a divisor of 15, 25, 30. 5 is the largest number that will divide them. Then we say 5 is the

greatest common divisor of 15, 25, 30. *The greatest common divisor of several numbers is the greatest number that will divide them.*

3. Find the greatest common divisor of 16, 24, 48, thus:

$$\begin{array}{r|lll} 2) & 16 & 24 & 48 \\ \hline 2) & 8 & 12 & 24 \\ \hline 2) & 4 & 6 & 12 \\ \hline & 2 & 3 & 6 \end{array}$$

$2\times2\times2=8$. 8 is the largest number that will divide 16, 24, 48.

4. Find the greatest common divisor of

20, 24, 36. 50, 70, 120. 72, 96, 144.
35, 42, 63. 18, 36, 64. 51, 57, 63.

5. What is a prime number?
Write the list of prime numbers up to 97.

6. Find the least common multiple of

18, 24, 30. 54, 63, 70.
35, 70, 105. 24, 72, 96.

7. Divide 303033 by 7895; by 8987.

LX.

1. Divide the product of $6\times18\times36\times48$ by the product of $32\times27\times42$. Arrange thus

$$\frac{6\times18\times36\times48}{32\times27\times42}$$

and use cancellation.

2. Write the definition of cancellation.

3. A vest is worth $\$2\frac{1}{2}$, a coat $\$9\frac{3}{8}$, pants $\$6\frac{2}{3}$. What is the value of the suit?

4. A coat required $4\frac{1}{4}$ yards of cloth, pants $2\frac{2}{3}$ yards, vest $\frac{7}{8}$ of a yard. Did the coat take more goods than the pants and vest? How much?

5. $15\frac{1}{2}-12\frac{3}{4}=?$ $225\frac{2}{3}-180\frac{7}{8}=?$ $444\frac{1}{8}-200\frac{3}{4}=?$

6. $3\frac{5}{9}+4\frac{6}{7}+5\frac{1}{8}=?$ $4\frac{7}{8}+6\frac{2}{3}+10\frac{1}{3}=?$

7. Multiply $\frac{3}{4}$ by $\frac{6}{7}$. $4\frac{1}{2}$ by 6. $\frac{7}{8}\times3\frac{1}{2}\times9=?$ 8 by $3\frac{1}{8}$.

8. $5\frac{1}{2}\div\frac{3}{4}=?$

$\frac{11}{2}\div\frac{3}{4}=\frac{22}{4}\div\frac{3}{4}=\frac{22}{3}=7\frac{1}{3}$. $\frac{11}{2}\div\frac{3}{4}=\frac{11}{2}\times\frac{4}{3}=\frac{22}{3}=7\frac{1}{3}$.

(The teacher will show the class how $7\frac{1}{3}$ is obtained by cancellation.)

Write rule for dividing fractions by second method.

9. Work by second method:

$7\frac{1}{3}\div\frac{2}{3}=?$	$8\frac{1}{4}\div\frac{3}{4}=?$	$\frac{5}{8}\div\frac{1}{2}=?$
$4\div\frac{1}{2}=?$	$9\frac{2}{3}\div7=?$	$\frac{5}{16}\div8=?$
$8\frac{2}{3}\div\frac{1}{3}=?$	$9\frac{3}{5}\div16=?$	$3\frac{1}{9}\div7=?$

LXI.

1. $\frac{8}{9}\div3=\frac{1}{9}$ $\frac{6}{9}\div3=\frac{2}{9}$ $\frac{12}{9}\div3=\frac{4}{9}$

$\frac{28}{10}\div4=?$ $\frac{16}{8}\div4=?$ $\frac{24}{3}\div6=?$

$\frac{10}{3}\div5=?$ $\frac{40}{6}\div5=?$ $\frac{50}{6}\div25=?$

2. A ton of coal is worth $\$5\frac{7}{16}$, a load of wood $\$3\frac{5}{12}$. How much more would two loads of wood cost than one ton of coal? *Ans.* $\$1\frac{19}{48}$.

3. If I put $\$40\frac{4}{5}$ in the bank, how much more must I put in that I may have there $\$65\frac{6}{7}$?

4. John earns $\$7\frac{3}{4}$ a week, Robert $\$6\frac{1}{3}$ dollars, James as much as both John and Robert. How much do they all earn?

5. Divide $7\times9\times15$ by 5×3.

6. I buy 7 pounds of tea at $\$\frac{4}{5}$ a pound, 15 lbs. of sugar at $\$\frac{1}{3}$ a pound. I hand the clerk $20; what change do I receive? *Ans.* $\$9\frac{2}{5}$.

LXII.

1. Fractions which have *"of"* between them are considered one fraction and are called *compound fractions.* Thus $\frac{1}{2}$ of $\frac{1}{4}$; $\frac{1}{3}$ of $\frac{5}{6}$; $\frac{2}{3}$ of $\frac{5}{8}$. *To simplify them, multiply their numerators for a new numerator—multiply their denominators for a new denominator.*

2. Reduce to simple fractions $\frac{5}{8}$ of $\frac{5}{9}$; $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{1}{4}$; $\frac{5}{8}$ of $\frac{7}{8} \times \frac{6}{10}$.

3. Here are some fractions that are different from any you have had.

$$\frac{\frac{2}{3}}{\frac{4}{5}} \qquad \frac{\frac{3}{4}}{\frac{5}{6}} \qquad \frac{2\frac{1}{2}}{3\frac{1}{3}}$$

They are called *complex fractions*, and have a fraction in the numerator, or in the denominator, or in both terms. The longest line in the fraction separates the real numerator from the real denominator.

$\frac{\frac{2}{3}}{\frac{4}{5}}$ is read $\frac{2}{3}$ divided by $\frac{4}{5}$.

$\frac{\frac{3}{4}}{\frac{5}{6}}$ is read $\frac{3}{4}$ divided by $\frac{5}{6}$.

The numerator is the dividend, the denominator is the divisor.

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$$

$$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{9}{10}$$

$$\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \times \frac{3}{10} = \frac{3}{4}$$

(The teacher will show the class how the result is obtained by cancellation.)

LXIII.

1. $\frac{1}{2}$ is equal to how many fourths? How many 6ths? How many 8ths? How many 10ths? How many 12ths? How many 16ths? How many 18ths? How many 24ths?

2. $\frac{1}{3}$ is equal to how many 6ths? How many 12ths? How many 15ths? How many 9ths? How many 18ths? How many 24ths? How many 30ths? How many 36ths?

3. $\frac{1}{4}$ is equal to how many 8ths? How many 12ths? How many 16ths? How many 24ths? How many 36ths?

4. $\frac{1}{5}$ is equal to how many tenths? How many 15ths? How many 25ths? How many 35ths? How many 45ths??

5. $\frac{1}{6}$ is equal to how many 12ths? How many 18ths? How many 24ths? How many 36ths? 48ths? 54ths?

6. $\frac{1}{7}$ is equal to how many fourteenths? How many 28ths? How many 35ths? 49ths? 63ds? 77ths?

7. $\frac{1}{8}$ is equal to how many 16ths? 24ths? 40ths? 56ths? 48ths? 64ths?

8. $\frac{1}{9}$ is equal to how many 18ths? 36ths? 54ths? 72ds? 99ths?

9. $\frac{1}{10}$ is equal to how many twentieths? 30ths? 40ths? 50ths? 70ths? 60ths? 80ths?

10. A had \$540, B had $\frac{1}{2}$ as much as A, C had as much as A and B, D had three times as much as A and C, E had twice as much as B and D, and F had $\frac{1}{2}$ as much as A, B, C, D, and E. What did each have?

11. How many cubic inches in $5\frac{1}{3}$ gallons?

LXIV.

1. *The design of the next eight lessons is to bring every principle of fractions before the pupil in each recitation.* Require the work to be brought to class on paper. At class have all the examples worked on the board by as many different pupils; *then rigidly require each example to be explained, after the following models.* Suppose the following be given: $\frac{14}{35}$; $5\frac{1}{3}$; $\frac{2}{3}+\frac{1}{4}$; $\frac{3}{5}\div\frac{2}{3}$.

The pupil, after working, will say: "*We are required to reduce* $\frac{14}{35}$ *to its lowest terms. This may be done by dividing both terms by* 7. $\frac{14}{35}=\frac{2}{5}$. $\frac{1}{5}$ of a unit equals $\frac{7}{35}$; $\frac{2}{5}$ equals $\frac{14}{35}$."

"*We are required to reduce* $5\frac{1}{3}$ *to an improper fraction. This may be done by multiplying the whole number* 5 *by the denominator* 3; *this gives* 15, *to which add the numerator* 1, *making* 16, *and under this place the denominator* 3. $5\frac{1}{3}=\frac{16}{3}$. $\frac{3}{3}$ in 1 unit; in 5 units there are $\frac{15}{3}$; $\frac{15}{3}+\frac{1}{3}=\frac{16}{3}$."

"*We are required to add* $\frac{2}{3}$ and $\frac{1}{4}$. *This may be done by reducing the fractions to other fractions having the same denominator. They may be reduced to twelfths:* $\frac{2}{3}$ *equals* $\frac{8}{12}$; $\frac{1}{4}$ *equals* $\frac{3}{12}$. $\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$."

"*We are required to divide* $\frac{3}{5}$ *by* $\frac{2}{3}$. *This may be done in one way by reducing the fractions to a common denominator, and then dividing the numerator of the dividend by the numerator of the divisor.* $\frac{3}{5}=\frac{9}{15}$; $\frac{2}{3}=\frac{10}{15}$; $\frac{9}{15}\div\frac{10}{15}=\frac{9}{10}$. *Or the division may be done by inverting the divisor, and by multiplying the numerators for a new numerator, and the denominators for a new denominator.* Thus: $\frac{3}{5}\div\frac{2}{3}=\frac{3}{5}\times\frac{3}{2}=\frac{9}{10}$."

1.	$\frac{16}{24}$	$\frac{25}{30}$	$\frac{48}{64}$
2.	$\frac{25}{8}$	$\frac{37}{12}$	$\frac{97}{25}$
3.	$3\frac{3}{4}$	$4\frac{4}{5}$	$10\frac{8}{9}$
4.	$\frac{2}{5}$	$\frac{3}{8}$	$\frac{4}{7}$

5. 3 to thirds. 4 to fifths. 6 to tenths.

6. $\frac{3}{4}=\frac{}{24}$ $\frac{5}{8}=\frac{}{48}$ $\frac{7}{9}=\frac{}{63}$

7. $\frac{3}{4}+\frac{3}{8}=?$ $\frac{5}{6}+\frac{7}{8}=?$ $\frac{1}{3}+\frac{3}{4}+\frac{5}{6}=?$

8. $\frac{3}{4}-\frac{2}{3}=?$ $\frac{7}{8}-\frac{2}{3}=?$ $\frac{5}{6}-\frac{3}{7}=?$

9. $\frac{2}{3}\times\frac{3}{4}=?$ $\frac{5}{7}\times\frac{5}{9}=?$ $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{7}=?$

10. $\frac{5}{9}\div\frac{3}{7}=?$ $\frac{6}{8}\div\frac{1}{3}=?$ $\frac{5}{7}\div\frac{1}{2}=?$

11. 7 is what part of 15? 9 is what part of 17?

12. $\frac{2}{3}$ of $45=?$ $\frac{3}{4}$ of $48=?$ $\frac{5}{6}$ of $42=?$

13. 12 is $\frac{3}{4}$ of ? 20 is $\frac{4}{5}$ of ? 30 is $\frac{5}{8}$ of ?

14. $\frac{\frac{3}{4}}{\frac{5}{8}}$ $\frac{2\frac{1}{4}}{5\frac{1}{3}}$ $\frac{3\frac{5}{16}}{\frac{3}{8}}$

LXV.

1. Write the list of prime factors up to 97.

2. Find the greatest common divisor of 36, 96, 144.

3. Find the least common multiple of 36, 96, 144.

4. $3\times18\times27\times45$, divided by $6\times9\times9\times15=?$ (Cancellation.)

5. $\frac{35}{85}$ $\frac{27}{63}$ $\frac{64}{144}$

6. $\frac{35}{9}$ $\frac{43}{12}$ $\frac{100}{30}$

7. $8\frac{5}{9}$ $9\frac{4}{5}$ $12\frac{6}{7}$

8. $\frac{3}{8}$ $\frac{5}{9}$ $\frac{7}{12}$

9. 5 to fourths. 6 to sevenths. 10 to thirds.

10. $\frac{5}{8}=\frac{}{48}$? $\frac{9}{10}=\frac{}{80}$? $\frac{9}{10}=\frac{}{90}$?

11. $\frac{2}{3}+\frac{6}{8}=?$ $\frac{3}{4}+\frac{5}{12}+\frac{7}{15}=?$ $2\frac{1}{2}+3\frac{1}{3}+4\frac{1}{4}=?$

12. $\frac{5}{6}-\frac{3}{5}=?$ $\frac{9}{10}-\frac{2}{5}=?$ $3\frac{1}{5}-2\frac{1}{2}=?$

13. $\frac{3}{8}\times\frac{5}{8}=?$ $\frac{4}{10}\times\frac{7}{8}\times\frac{15}{21}=?$ $1\frac{1}{2}\times3\frac{1}{3}\times\frac{2}{15}=?$

14. $\frac{3}{7}\div6=?$ $\frac{7}{9}\div\frac{1}{3}=?$ $\frac{8}{7}\div2=?$

15. 12 is what part of 21? 11 is what part of 16?

16. $\frac{3}{8}$ of $64=?$ $\frac{5}{6}$ of $72=?$ $\frac{8}{9}$ of $81=?$

17. 21 is $\frac{3}{8}$ of what number? 45 is $\frac{5}{9}$ of what number?

18. $\frac{2\frac{1}{3}}{5\frac{3}{5}}$ $\frac{\frac{7}{12}}{4}$

LXVI.

1. Find the greatest common divisor of 39, 51, 87.
2. Find the least common multiple of 15, 45, 70, 90.
3. $4\times28\times35\times9$ divided by $56\times70\times27$. (Cancellation.)
4. $\frac{2}{10}$ to fifths. $\frac{8}{24}$ to fourths. $\frac{21}{42}$ to 6ths.
5. $\frac{30}{7}$ to 1's. $\frac{47}{4}$ to 1's. $\frac{45}{9}$ to 1's.
6. $5\frac{1}{3}$ to thirds. $28\frac{3}{5}$ to fifths. $16\frac{4}{15}$ to fifteenths.
7. 6 to halves. 6 to fourths. 6 to ninths.
8. $\frac{3}{4}$ to 12ths. $\frac{2}{9}$ to 45ths. $\frac{2}{9}$ to 54ths.
9. Find the sum and difference of $\frac{1}{2}$ and $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{4}$, $\frac{1}{9}$ and $\frac{1}{3}$.
10. $\frac{1}{2}$ of $\frac{4}{7}$=? $\frac{1}{3}$ of $\frac{15}{20}$=? $\frac{1}{8}$ of $\frac{3}{5}$ of $2\frac{1}{2}$=?
11. $\frac{6}{7}\times\frac{21}{28}$=? $\frac{16}{21}\times\frac{7}{8}$=? $64\times\frac{9}{16}$=?
12. $3\div\frac{1}{5}$=? $12\div\frac{1}{12}$=? $2\div\frac{1}{9}$=?
13. $\frac{4}{5}$ of 500=? $\frac{3}{4}$ of 1200=?
14. 50 is $\frac{5}{9}$ of what number? 50 is $\frac{10}{11}$ of what number?
15. $\frac{\frac{3}{10}}{\frac{20}{40}}$ $\qquad$ $\frac{5\frac{1}{2}}{3\frac{1}{8}}$

LXVII.

1. What is the difference between $\frac{3}{4}$ of 40 and $\frac{4}{5}$ of 30?
2. What part of 4 is 1? Of 4 is 3?
3. \$4 is what part of \$5? of \$7? of \$9? of \$11?
4. How many 8ths in $4\frac{5}{8}$? How many 7ths in $3\frac{6}{7}$?
5. How many 1's in $\frac{63}{10}$? How many 1's in $\frac{84}{7}$?
6. 12 to 5ths. 12 to 7ths.
7. $\frac{5}{6}$ to 18ths. $\frac{5}{9}$ to 18ths.
8. $\frac{24}{60}$ to 5ths. $\frac{3}{4}$ and $\frac{5}{6}$ to 12ths.
9. Find the sum, difference, product, and quotient of $3\frac{5}{12}$ and $\frac{3}{7}$.
10. $\frac{1}{3}$ of 5=? $\frac{1}{4}$ of 9=? $\frac{1}{6}$ of 11=?

11. $\frac{5}{6}$ of 600 = ? $\frac{5}{6}$ of 540 = ? $\frac{5}{6}$ of 84 = ?

12. 80 is $\frac{4}{7}$ of what number? 14 is $\frac{2}{7}$ of what number?

13. $\frac{5}{2\frac{1}{2}}$ $\frac{7}{2\frac{1}{3}}$

(Teachers should rigidly follow models in Lesson LXII.)

LXVIII.

1. $\frac{9}{18}$, $\frac{20}{30}$, $\frac{25}{75}$ to lowest terms.
2. $\frac{5}{7}$, $\frac{5}{11}$, $\frac{5}{18}$ to higher terms.
3. $\frac{1}{3}$ and $\frac{4}{5}$, $\frac{3}{8}$ and $\frac{1}{3}$, $\frac{3}{7}$ and $\frac{3}{5}$, $\frac{4}{15}$, $\frac{1}{12}$, and $\frac{1}{6}$ to fractions having a common denominator.
4. Add $1\frac{1}{2}$, $2\frac{1}{4}$, and $\frac{5}{8}$.
5. $\frac{1}{2} - \frac{4}{9} = ?$ $4\frac{3}{7} - \frac{7}{9} = ?$ $16 - 9\frac{3}{4} = ?$ $28 - 17\frac{3}{8} = ?$
6. 2 times $4\frac{1}{2} = ?$ 3 times $5\frac{1}{3} = ?$ 4 times $4\frac{4}{5} = ?$
   6 times $5\frac{1}{7} = ?$ 8 times $4\frac{5}{9} = ?$ 7 times $3\frac{2}{3} = ?$
7. $\frac{1}{2}$ of $\frac{1}{2} = ?$ $\frac{1}{2}$ of $\frac{1}{5} = ?$ $\frac{1}{4}$ of $\frac{1}{5} = ?$
   $\frac{1}{3}$ of $\frac{1}{4} = ?$ $\frac{7}{5}$ of $\frac{2}{3} = ?$ $\frac{1}{8}$ of $\frac{1}{6}$ of $48 = ?$
8. $\frac{7}{8} \div \frac{2}{5} = ?$ (Dividend.) $\frac{1}{2}$ of $\frac{5}{9} \div \frac{3}{4} = ?$ (Dividend.) (Divisor.) $\frac{2}{3}$ of $\frac{4}{5} \div \frac{1}{4}$ of $\frac{2}{7}$ (Divisor.)
   $1\frac{3}{11} \div \frac{1}{2}$ of $\frac{2}{3} = ?$

   What are the terms in division called?

   Which term is placed before the division sign?
9. $\frac{\frac{5}{8}}{\frac{10}{24}}$ $\frac{6\frac{2}{3}}{4\frac{2}{7}}$ $\frac{4\frac{4}{5}}{6}$

LXIX.

1. Write the prime numbers up to 97.
2. Find the greatest common divisor of 45, 75, 105.
3. Find the least common multiple of 45, 75, 105.

   A multiple is a number made by multiplying other numbers together. Thus: $2 \times 3 \times 5 = 30$. 30 is a multiple of 2, 3, and 5. 2, 3, and 5 are factors, or makers, or divisors of 30.

4. $\frac{75}{100}$ $\frac{25}{5}$ to 1's. $12\frac{1}{3}$

$\frac{56}{48}$ $\frac{72}{9}$ to 1's. $7\frac{4}{5}$

$\frac{81}{100}$ $\frac{120}{12}$ to 1's. $10\frac{3}{8}$

$\frac{45}{90}$ $\frac{100}{5}$ to 1's. $15\frac{3}{4}$

5. How many units in $\frac{24}{10}$? $\frac{45}{7}$? $\frac{146}{12}$?

6. $\frac{3}{7}+\frac{1}{4}+\frac{3}{14}=?$ $\frac{5}{6}$, $\frac{4}{9}$, $\frac{1}{6}$ to 18ths. $\frac{3}{10}$, $\frac{7}{20}$, $\frac{9}{30}$ to 60ths. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=?$

7. $3\frac{1}{3}-2\frac{1}{2}=?$ $12\frac{1}{6}-8\frac{3}{4}=?$

8. $8\frac{1}{3}\div 5=?$ $10\frac{2}{7}\div 7=?$ $18\frac{1}{2}\div 12=?$

9. $\frac{\frac{1}{2}\text{ of }\frac{2}{3}}{4\frac{1}{2}}$ $\frac{\frac{2}{3}\text{ of }\frac{2}{5}}{\frac{7}{15}}$ $\frac{\frac{1}{2}\text{ of }\frac{1}{8}\text{ of }\frac{1}{3}}{\frac{13}{48}}$

LXX.

1. Divide 20×28 by $7\times 5\times 4$.

Write the definition of cancellation.

2. Divide $7\times 15\times 24$ by 14×4.

3. *Reduction of a denominate number is the process of changing its denomination without altering its value.*

Reduction of a fraction consists in changing its form without altering its value.

4. Reduce $\frac{54}{6}$, $\frac{96}{8}$, $\frac{175}{9}$, $\frac{225}{18}$.

5. Reduce 8 to 5ths. 9 to 10ths. 9 to 12ths. 14 to 7ths.

6. $\frac{5}{9}$ to 54ths. $\frac{7}{9}$ to 45ths.

7. How many fourths in $\frac{1}{2}$? how many eighths?

8. In $\frac{1}{3}$ how many 6ths? how many 9ths?

9. $\frac{8}{9}=\frac{}{72}$? $\frac{5}{12}=\frac{}{96}$? $\frac{5}{12}=\frac{}{84}$?

10. $\frac{3}{15}$, $\frac{3}{7}$, and $\frac{3}{5}$ to other fractions having 105 for their denominator.

11. 5 times $\frac{3}{5}=?$ 9 times $\frac{3}{4}=?$ 12 times $\frac{5}{8}=?$

12. What part of 45 cents is 7 cents? is 9 cents? is 17 cents? is 29 cents?

(The teacher should bring the cents before the class.)

13. In $\frac{1}{5}$ how many tenths? how many fifteenths?

LXXI.

1. 2 times $5\frac{3}{8}$=? 10 times $6\frac{3}{4}$=? 12 times $2\frac{2}{3}$=?
   8 times $3\frac{1}{3}$=? 9 times $5\frac{1}{8}$=? 11 times $3\frac{2}{3}$=?
2. Find the sum, difference, product, and quotient of $9\frac{1}{3}$ and $5\frac{2}{7}$.
3. $\frac{15}{17}\div 5$=? $\frac{16}{17}\div 8$=? $\frac{20}{21}\div 5$=?
4. $15\frac{2}{3}+18\frac{3}{4}+25\frac{5}{9}$=?
5. $\frac{9}{16}+24\frac{5}{6}+\frac{7}{8}$=?
6. $\frac{5}{8}$ of 480=? $\frac{7}{9}$ of 630=? $\frac{3}{4}$ of 2400=?
7. 48 is $\frac{3}{5}$. 63 is $\frac{7}{9}$. 72 is $\frac{8}{9}$.
8. Write the complete Table of Principles.
9. What is reduction of denominate numbers?
10. What is reduction of fractions?

LXXII.

1. If $\frac{3}{7}$ of an acre of land sell for \$21, what will an acre sell for?
2. If I earn $\$\frac{7}{8}$ to-day, $\$2\frac{1}{4}$ to-morrow, $\$1\frac{5}{12}$ next day; what will be the sum of my earnings in the three days? *Ans.* $\$4\frac{1}{2}\frac{3}{4}$.
3. If 4 books are worth $\$\frac{8}{9}$, what is 1 book worth?
4. How many times is $\frac{1}{3}$ contained in 4?
5. How many times is $\frac{1}{5}$ contained in 5?
6. What will $3\frac{3}{4}$ dozen of eggs cost at $12\frac{1}{2}$ cents a dozen?
7. If a man cut $\frac{7}{8}$ of a cord of wood in 1 day, how much can he cut in $4\frac{1}{2}$ days?
8. A grocer drew from a hogshead of molasses, $15\frac{3}{4}$ gallons at one time and $27\frac{4}{5}$ at another; how much remained in the hogshead? *Ans.* $19\frac{9}{20}$ gal.
9. Which would give more pay, to work 3 days at \$2 a day, or to get $\$1\frac{1}{3}$ the first day, $\$\frac{7}{8}$ the second, and $\$1\frac{1}{2}$ the third day?

LXXIII.

1. At $\$\frac{3}{4}$ a bushel, how much will $\frac{9}{10}$ of a bushel cost?

2. If $2\frac{1}{2}$ lbs. of coffee cost $\$\frac{15}{16}$, what will 1 pound cost?

3. If $\$\frac{8}{9}$ be divided among 6 boys, what will each boy receive?

4. Which cost more, 12 yards of cloth at $\$\frac{3}{4}$ a yard, or 5 pounds of tea at $\$\frac{7}{8}$ a pound?

5. *If the price of one yard, or one bushel, or one pound is given, you multiply. If the price of one yard, or one bushel, or one pound is asked, you divide.*

6. If $\frac{5}{8}$ of a ton of iron is worth $44, what is one ton worth?

7. If a man travel $\frac{3}{4}$ of a mile in 1 hour, how many hours will it take him to travel 12 miles?

8. If 1 quart of nuts is worth $\frac{2}{3}$ of a dime, what must you pay for 4 pecks? *Ans.* $21\frac{1}{3}$ dimes.

9. What is the cost of 5 gallons of milk at $\frac{1}{4}$ of a dime for 1 pint?

LXXIV.

1. Mary paid $\$2\frac{1}{3}$ for a dress, $\$1\frac{3}{4}$ for shoes, and $\$\frac{1}{3}$ for ribbon. If she gave the merchant $5, how much change did she receive? *Ans.* $\$\frac{7}{12}$.

2. From 1 gallon of molasses $\frac{7}{8}$ of a quart was used, how much was left?

3. Three men own a house; one owns $\frac{1}{5}$ of it, another owns $\frac{2}{7}$ of it; what does the third man own?

4. What number must be taken from 36 that the remainder may be $7\frac{6}{7}$?

5. If the dividend is $\frac{9}{10}$ and the divisor $\frac{2}{3}$, what is the quotient?

6. If the quotient is $2\frac{1}{2}$ and the divisor $\frac{7}{8}$, what is the dividend?

7. If a boy gave each of his 4 companions $\frac{3}{8}$ of a quart of chestnuts, and had $4\frac{1}{2}$ quarts left, how many quarts had he at first?

8. How can you tell proper fractions from improper fractions?

9. If 1 horse eat $\frac{3}{8}$ of a ton of hay in 1 month, how much will 20 horses eat in 5 months?

LXXV.

1.

	mi.	rds.	yds.	ft.	in.
	17	70	2	1	10
	16	30	1	1	9
	25	16	1	2	11

	mi.	rds.	yds.	ft.	in.
From	17	60	1	1	7
Take	14	70	2	2	9

	mi	rds.	yd.	ft.	in.	
2. Multiply	25	16	1	2	11	by 9.

3. Divide 12 hhds., 16 gal., 2 qts, 1 pt., 1 gi. by 43.

4. Divide 17 mi., 70 rds., 2 yds., 1 ft., 10 in. by 5.

5. Change 5000 pounds to higher denominations.

6. Reduce 1540 pints to higher denominations.

7. Reduce 2500 grains to higher denominations.

8. How many bushels (8 gallons to the bushel) can be contained in a box 4 feet long, 3 feet wide, and 2 feet deep? *Ans.* $22\frac{3}{7}\frac{4}{7}$ bushels.

How many cubic inches in 1 gallon?

(The teacher may regard this as the close of five months' work.)

LXXVI.

The pupils should commit the following diagram to memory, and they are the smart boys and girls who will do this in one day:

FRACTIONS.

Common.

Common fractions are made by dividing the *unit* into *any* number of equal parts. Thus: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{8}{10}$, $\frac{9}{11}$, $\frac{7}{12}$, $\frac{5}{13}$, $\frac{6}{14}$, $\frac{7}{15}$, $\frac{10}{18}$, $\frac{11}{17}$, $\frac{12}{18}$, $\frac{13}{19}$, $\frac{9}{20}$, and so on. Therefore *common fractions* may have *any* number for a denominator.

Decimal.

Decimal fractions are made by dividing the *unit* into 10 equal parts, called tenths; these tenths into ten other equal parts, called hundredths; these hundredths into ten other equal parts, called thousandths; and so on. Thus: $\frac{1}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, $\frac{1}{100}$, $\frac{8}{100}$, $\frac{15}{100}$, $\frac{25}{100}$, $\frac{38}{100}$, $\frac{1}{1000}$, $\frac{11}{1000}$, $\frac{15}{1000}$, $\frac{135}{1000}$, $\frac{1}{10000}$, $\frac{25}{10000}$, $\frac{225}{10000}$, $\frac{146}{10000}$. Therefore a decimal fraction must have for its denominator 10, or 100, or 1000, or 10000, etc.

LXXVII.

1. How many of our young friends have committed Lesson LXXVI. to memory, and can place it on the board without the book? We hope all of them can.

Now commit the following paragraph to memory.

2. *Decimal fractions are so peculiar that we can write and use them by writing the numerator only. But in this case the decimal will have as many figures as* THERE ARE CIPHERS IN THE DENOMINATOR.

3. Now you must remember too, that *all* fractions are common fractions, and we give the name decimal fractions only to these *peculiar* common fractions.

4. Let us write some of them, and do not fail to

place a point (.) before the numerator or decimal when you omit the denominator:

$\frac{1}{10}$=.1 $\frac{1}{100}$=.01 $\frac{65}{100}$=.65
$\frac{2}{10}$=.2 $\frac{2}{100}$=.02 $\frac{84}{100}$=.84
$\frac{3}{10}$=.3 $\frac{9}{100}$=.09 $\frac{96}{100}$=.96
$\frac{5}{10}$=.5 $\frac{15}{100}$=.15 $\frac{105}{100}$=1.05
$\frac{9}{10}$=.9 $\frac{25}{100}$=.25 $\frac{124}{100}$=1.24
$\frac{11}{10}$=1.1 $\frac{38}{100}$=.38 $\frac{235}{100}$=2.35
$\frac{12}{10}$=1.2 $\frac{42}{100}$=.42 $\frac{315}{100}$=3.15
$\frac{15}{10}$=1.5 $\frac{53}{100}$=.53 $\frac{417}{100}$=4.17

LXXVIII.

1. Let me remind you that the number of figures on the right of the decimal point (.) will always be equal to the ciphers in the denominator. Copy the following, and then write them again without looking at the second and fourth columns:

$\frac{1}{1000}$=.001 $\frac{1}{10000}$=.0001
$\frac{3}{1000}$=.003 $\frac{2}{10000}$=.0002
$\frac{7}{1000}$=.007 $\frac{7}{10000}$=.0007
$\frac{9}{1000}$=.009 $\frac{10}{10000}$=.0010
$\frac{15}{1000}$=.015 $\frac{15}{10000}$=.0015
$\frac{26}{1000}$=.026 $\frac{115}{10000}$=.0115
$\frac{35}{1000}$=.035 $\frac{345}{10000}$=.0345
$\frac{135}{1000}$=.135 $\frac{754}{10000}$=.0754
$\frac{344}{1000}$=.344 $\frac{1225}{10000}$=.1225
$\frac{525}{1000}$=.525 $\frac{2500}{10000}$=.2500
$\frac{650}{1000}$=.650 $\frac{7854}{10000}$=.7854
$\frac{875}{1000}$=.875 $\frac{17854}{10000}$=1.7854
$\frac{1875}{1000}$=1.875 $\frac{25235}{10000}$=2.5235
$\frac{2525}{1000}$=2.525 $\frac{37054}{10000}$=3.7054
$\frac{3874}{1000}$=3.874 $\frac{98009}{10000}$=9.8009

LXXIX.

The pupil will copy the following lists of fractions and write the decimal after each one:

1. $\frac{5}{10}$=.5 $\frac{9}{10}$= 9 $\frac{15}{1000}$=.015 $\frac{5}{1000}$=.005
   $\frac{5}{100}$=? $\frac{19}{10}$=? $\frac{175}{1000}$=? $\frac{35}{1000}$=?
   $\frac{7}{10}$=? $\frac{19}{100}$=? $\frac{644}{1000}$=? $\frac{335}{1000}$=?
   $\frac{15}{100}$=? $\frac{119}{100}$=? $\frac{1644}{1000}$=? $\frac{1335}{1000}$=?
   $\frac{35}{100}$=? $\frac{35}{100}$=? $\frac{1864}{1000}$=? $\frac{2}{10}$=?
   $\frac{35}{10}$=? $\frac{135}{100}$=? $\frac{6}{1000}$=? $\frac{12}{10}$=?
   $\frac{135}{100}$=? $\frac{225}{100}$=? $\frac{12}{100}$=?
2. $\frac{35}{10000}$=.0035 $\frac{5}{100000}$=.00005 $\frac{1375}{10000000}$=?
   $\frac{356}{1000}$=? $\frac{155}{100000}$=? $\frac{2544}{10000000}$=?
   $\frac{356}{100}$=? $\frac{375}{100000}$=? $\frac{3755543}{10000000}$=?
   $\frac{78}{10000}$=? $\frac{279}{10000}$=? $\frac{1279}{10000}$=?
3. $\frac{3}{1000000}$=? $\frac{135}{1000000}$=? $\frac{2574}{1000000}$=?
   $\frac{3}{10}$=? $\frac{1135}{1000000}$=? $\frac{12509}{1000000}$=?
   $\frac{25}{100}$=? $\frac{45}{1000}$=? $\frac{3756554}{1000000}$=?
   $\frac{25}{10000}$=? $\frac{25}{1000}$=? $\frac{25}{100}$=?
4. $\frac{7}{10}$=? $\frac{27}{100}$=? $\frac{35}{100}$=?
   $\frac{7}{1000}$=? $\frac{127}{10000}$=? $\frac{335}{1000}$=?
   $\frac{17}{100000}$=? $\frac{127}{1000000}$=? $\frac{14}{10}$=?
   $\frac{38}{1000000}$=? $\frac{339}{1000000}$=? $\frac{339}{10}$=?

LXXX.

Write the common fractions after these decimals:

.5=$\frac{5}{10}$	.05=$\frac{5}{100}$	.39=?
.9=$\frac{9}{10}$	.25=$\frac{25}{100}$	.10=?
1.9=$\frac{19}{10}$	.27=?	.01=?
.7=?	.33=?	.09=?
.3=?	.87=?	.04=?
1.3=?	.55=?	.44=?
.1=?	1.55=?	2.44=?
1.1=?	2.25=?	18.75=?

2.	.003=?	.435=?	1.225=?
	.006=?	.605=?	3.313=?
	.015=?	.725=?	6.414=?
	.025=?	.333=?	7.252=?
	.017=?	.910=?	.018=?
	.117=?	.811=?	.025=?
	.225=?	.805=?	1.025=?
3.	.0005=$\frac{5}{10000}$	.00004=$\frac{4}{100000}$	.01875=?
	.0025=?	.00025=?	.22313=?
	.0126=?	.00137=?	.70177=?
	.0375=?	.00337=?	.84101=?
	.1375=?	.00485=?	.76034=?

(The teacher should now require the pupils to read aloud in class Lessons LXXVIII., LXXIX., and LXXX.

LXXXI.

1. All the figures on the right of the decimal point are called *the decimal* or *so many decimal places.*

How many decimal places does it require to write tenths? To write hundredths? To write thousandths? To write millionths? To write ten-thousandths? To write hundred-thousandths?

2. Fill out these columns with the right fractions and decimals:

$\frac{5}{10}$=?		?=.25
?=.6		$\frac{38}{100}$=?
$\frac{5}{10}$=?		?=.16
?=.12		$\frac{45}{10}$=?
$\frac{18}{100}$=?		?=.75
3. $\frac{85}{100}$=?	?=1.45	$\frac{26}{100}$=?
?=.64	$\frac{37}{100}$=?	?=1.26
$\frac{125}{100}$=?	?=.28	$\frac{34}{100}$=?
?=1.35	$\frac{29}{100}$=?	?=.11
$\frac{644}{100}$=?	?=1.29	$\frac{22}{100}$=?

4. $\frac{25}{1000}$ = ? $\frac{915}{1000}$ = ? $\frac{18}{100}$ = ?

? = .017 ? = 1.205 ? = .77

$\frac{125}{1000}$ = ? $\frac{1875}{100}$ = ? $\frac{177}{100}$ = ?

? = .305 ? = 2.316 ? = 3.05

$\frac{405}{1000}$ = ? $\frac{2750}{1000}$ = ? $\frac{675}{1000}$ = ?

? = .675 ? = 3.319 ? = 4.44

5. How many decimal places does it take to write tenths? To write thousandths? To write hundredths? To write ten-thousandths? To write millionths? To write hundred-thousandths?

6. The pupil will now notice how easy it is to add, subtract, multiply, and divide decimals.

Add five tenths and sixty-five hundredths.

Add .5	Add .37	Add .25
.65	1.95	.325
1.15	.345	.605
	2.665	1.180

From .9	From .95	From .875	From .75
Take .5	Take .37	Take .659	Take .486
.4	.58	.216	.264

Multiply .5	Multiply .65	Multiply .0008
By 4	By 4	By 4
2.0	2.60	.0032

9. 4)2.64 264 hundredths divided by 4 gives 66 hundredths.
.66

LXXXII.

1. When you add or subtract decimals, be careful to place a decimal point in the answer under the points above.

.65	8.7	.35	8.15
.5	3.85	3.62	7.05
.375	.45	.017	7.005
.6085	2.015	4.045	13.018

2. 1.35 .024 3.33 2.2
.047 15.024 33.033 .22
.355 18.137 3.333 .666
8.51 .654 .4 5.5055
9.019 .099 .55 .89

3. From 9.09 15.8—6.9=? 2.40—.024=?
Take 7.75 8.75—3.29=? 26.275—19.49=?
$105—$.875=?

4. Multiply 16.5 18.24 18.24 6.05
By 5 6 36 48

5. 85.5÷5=? 109.44÷6=? 36.114÷6=?
6. 3.30÷6=? 2.485÷7=? 53.28÷8=?

(The teacher should now have the decimals in the last three lessons read aloud in class.)

LXXXIII.

1. How many decimal places are required to write tenths? To write thousandths? To write hundredths? To write ten-thousandths? To write millionths? To write hundred-thousandths?

2. 75.075 60.60 27.27
63.0225 6.066 35.035
.57 6.0775 5.175
4.075 333.014 18.025
.922 98.98 .3765

3. From 22.075 From 335.335
Take 19.198 Take 197.127

4. 4.65×24=? 46.125×34=? 56.375×45=?
5. 237.03×54=? 174.065×67=? 8.95×78=?
6. 1896.24÷8=? 1218.455÷7=? 80.55÷9=?
7. 4.100÷4=? 38.484÷6=? 16.565÷5=?

8. A man had $7200; he gave his oldest son $\frac{1}{9}$ of it, his youngest son $\frac{1}{8}$ of the remainder, his oldest daughter $\frac{1}{7}$ of the remainder, his youngest daughter $\frac{1}{6}$ of the remainder, his wife $\frac{3}{5}$ of the remainder; the balance he paid for a house. What did each receive, and what did the house cost?

Ans. $1600 for house.

LXXXIV.

1. $\frac{5}{10}=.5$ $\frac{35}{1000}=?$ $\frac{55}{10000}=?$ $\frac{7}{1000000}=?$
   $\frac{25}{100}=25$ $\frac{84}{1000}=?$ $\frac{805}{10000}=?$ $\frac{601}{1000000}=?$
   $\frac{37}{100}=?$ $\frac{384}{1000}=?$ $\frac{1805}{10000}=?$ $\frac{395}{1000000}=?$
   $\frac{137}{100}=?$ $\frac{275}{1000}=?$ $\frac{375}{100000}=?$ $\frac{1395}{1000000}=?$
   $\frac{29}{100}=?$ $\frac{673}{1000}=?$ $\frac{1375}{100000}=?$ $\frac{40765}{1000000}=?$
2. $.4=\frac{4}{10}$ $.36=?$ $2.36=?$ $.76=?$
   $.6=\frac{6}{10}$ $.036=?$ $15.036=?$ $.0852=?$
   $.16=\frac{16}{100}$ $.072=?$ $5.5=?$ $.09375=?$
   $.25=?$ $.0375=?$ $9.09=?$ $.1145=?$
   $.225=?$ $.44=?$ $.099=?$ $.014=?$
   $.045=?$ $.86=?$ $.6075=?$ $.067=?$

3. Write from memory the diagram of fractions, exactly as it is in Lesson LXXVI.

LXXXV.

1. We now give the pupil two rules, and he must not fail to commit them to memory at once.

   To multiply decimals: *Multiply as in whole numbers, and from the right of the product point off as many places for decimals as there are decimal places in the multiplicand and multiplier.*

   If there be not as many figures in the product as you need to point off, prefix ciphers to the product.

$4.\overline{575}$	$.\overline{371}$
$3.\overline{25}$	$.\overline{025}$
22875	1855
9150	742
13725	.009275
14.86875	

You see a bar over the decimal. Counting the figures under the bars, there are five to point off in the product in the first example. In the second example there are six decimal places under the bars. As you have only four figures in the product, prefix two ciphers.

2. $5.\overline{35}\times6.\overline{5}=?$ $35.\overline{875}\times.\overline{075}=?$ $4.\overline{85}\times75=?$
$6.\overline{46}\times.\overline{375}=?$ $17.\overline{067}\times.\overline{125}=?$ $28.\overline{28}\times1.\overline{5}=?$
$.\overline{385}\times2.\overline{5}=?$ $18.\overline{001}\times.\overline{75}=?$ $36.\overline{15}\times14=?$

LXXXVI.

1. Here is the other rule, which you will learn at once.

To divide decimals: *Divide as in whole numbers. Point off from the right of the quotient as many decimal places as the number of decimals in the dividend is greater than the number of decimals in the divisor.*

If there be not as many figures in the quotient as you need to point off, prefix ciphers. Remember, too, that when there are more decimals in the divisor than in the dividend, you must annex ciphers to the dividend before you divide.

$6.\overline{7288}\div64.\overline{7}$

$64.\overline{7}\,)\,6.\overline{7288}\,(\,.104$

647	64.7	Proof.
2588	728	
2588	416	
	624	
	6.7288	

You see there are four decimal places in the dividend and one in the divisor. One from four leaves three. Point off three in the quotient.

1. $.\overline{01183} \div 11.\overline{83} = ?$ $.\overline{2565} \div .\overline{45} = ?$
2. $.\overline{2565} \div .\overline{57} = ?$ $13.\overline{824} \div 1.\overline{44} = ?$
3. $13.\overline{824} \div 9.\overline{6} = ?$ $29.\overline{13305} \div 42.\overline{53} = ?$
4. $29.\overline{13305} \div .\overline{685} = ?$ $232.\overline{725} \div 3.\overline{21} = ?$

Prove every one of these examples.

LXXXVII.

1. $5.305 \times 6.4 = ?$ $3.75 \times 2.84 = ?$
2. $16.33 \times 3.14 = ?$ $18.25 \times .875 = ?$
3. $37.85 \times .476 = ?$ $29.05 \times 39.1 = ?$
4. $25.016 \times .38 = ?$ $.0375 \times .0895 = ?$

Prove your work in every one of these examples, and mark every term. Thus:

	Divisor.	Dividend.	Quotient.
5.305	5.$\overline{305}$)	33.$\overline{9520}$ (	6.4
6.4		31830	
21220		21220	
31830		21220	
Product 33.9520			

LXXXVIII.

1. $.5184 \div .72 = ?$ $.23328 \div .36 = ?$
2. $.23328 \div .648 = ?$ $62.1351 \div 2.43 = ?$
3. $87.912 \div 2.16 = ?$ $87.912 \div 40.7 = ?$
4. $408.591 \div 99.9 = ?$ $.290752 \div .704 = ?$

Prove your work in every one of these examples.

LXXXIX.

1. Find the sum of twenty-five hundredths, twen-

ty-five tenths, one hundred and twenty-five hundredths, twenty-five thousandths, 25 millionths.

2. Find the difference between 5 tenths and 10 hundredths; between 7 hundredths and seven thousandths.

3. 6.25—.75=? 3.712—1.87=?

4. $3.875—$.893=? .0814—9 thousandths=?

5. 14.095—8.759=? .025—37 ten-thousandths=?

6. Write the following expressions entirely in figures, both as common fractions and decimals: 5 tenths; 18 hundredths; 29 thousandths; 145 millionths; 295 ten-thousandths; 484 hundred-thousandths; 9 tenths; 75 hundredths.

7. 64.125×68.75 Prove.

8. .47502÷.609 Prove.

XC.

1. Write the rule for multiplication of decimals.

2. Write the rule for division of decimals.

3. Write from memory the diagram of fractions in Lesson LXXVI.

4. Write the two ways of dividing common fractions. Make an example and work it by both methods.

5. Write the list of prime numbers up to 97.

6. Find the prime factors of 144.

A factor is a divisor; hence you must divide by prime numbers to get prime factors. Thus:

$$
\begin{array}{r}
2)\underline{144} \\
2)\underline{72} \\
2)\underline{36} \\
2)\underline{18} \\
3)\underline{9} \\
3
\end{array}
$$

The prime factors of 144 are:

$$2\times2\times2\times2\times3\times3.$$

7. Find the prime factors of 248; of 96.

XCI.

1. You are again reminded that *when the divisor has more decimal places than the dividend you must annex ciphers to the dividend to make its decimals equal to, or more than those in the divisor, before you divide.*

2. $7\div.035=?$ $7\div.0035=?$ $7\div.35=?$

3. $6\div.8=?$ $6\div.08=?$ $6\div.004=?$

4. $100\div.008=?$ $100\div.4=?$ $100=.05=?$

5. $7805\div.035$

Prove each example, and you will know whether your divison is correct. Thus:

$7\div.035=?$ $.035)7.000(200$ 200 Proof.

70 .035

00 1000

600

7.000

You notice the dividend, 7, has no decimals. By giving it three decimals it becomes 7000 thousandths. You may then divide.

XCII.

1. $535.8\div9.4=?$ $301.55\div815=?$

2. $301.68\div.36=?$ $3112.49272\div4.078=?$

3. $6.51021\div2.03=?$ $60.726\div.087=?$

4. $1602\div.036=?$ $1000\div.025=?$

Prove every example.

XCIII.

1. Write from memory the rule for multiplication of decimals. Make your own example and explain the pointing off.

2. Write from memory the rule for division of decimals. Illustrate this rule with the example you have just made.

3. $5451 \div .69 = ?$ $48.085 \div 59 = ?$

4. $2681.25 \div 715 = ?$ $1427.88 \div 438 = ?$

5. $33328.15 \div 6.45 = ?$ $23.328 \div 36 = ?$

6. From 10 take one-tenth. Subtract .05 from 84 thousandths. From 40 take 40 hundredths. From 500 take 500 tenths. $100 - .01 = ?$ $20 - .09 = ?$

XCIV.

1. Write from memory the two ways of performing division of common fractions.

Work each one of the following examples both ways: $\frac{5}{16} \div \frac{1}{8} = ?$ $\frac{9}{10} \div \frac{2}{3} = ?$ $\frac{7}{12} \div \frac{2}{5} = ?$

$\frac{5}{16} \div \frac{1}{8} = \frac{5}{16} \div \frac{2}{16} = \frac{5}{2} = 2\frac{1}{2}$ $\frac{5}{16} \div \frac{1}{8} = \frac{5}{16} \times \frac{8}{1} = \frac{5}{2} = 2\frac{1}{2}$

(The teacher will show the class how the $2\frac{1}{2}$ is obtained by cancellation.)

2. $1134.6 \div 732 = ?$ $1134.6 \div 1.55 = ?$

3. $6.94848 \div 1.034 = ?$ $6.94848 \div 6.72 = ?$

Prove each example.

4. Write from memory the rules for multiplication and division of decimals.

XCV.

1. You are sometimes required to change common fractions to decimals, or decimals to common fractions.

Change $\frac{1}{2}$ to a decimal. $\frac{1}{2}$ must be changed to tenths, or hundredths, etc. Multiply both terms by 5. $\frac{1}{2} = \frac{5}{10} = .5$

$\frac{1}{4}$ to a decimal. Multiply both terms of $\frac{1}{4}$ by 25. $\frac{1}{4} = \frac{25}{100} = .25$

$\frac{3}{8}$ to a decimal. Multiply both terms of $\frac{3}{8}$ by 125. $\frac{3}{8}=\frac{375}{1000}=.375$

Here is a convenient way: Annex decimal ciphers to the numerator, divide by the denominator, point off in the quotient as many decimals as are equal to the number of ciphers annexed.

$\frac{1}{2}$)1.0 Change the numerator 1 to 1.0 (ten
.5 tenths), 1.0 divided by 2 gives .5 (five tenths).

$\frac{1}{4}$)1.00 Change the numerator 1 to hundredths.
.25 1.00 divided by 4 gives .25

2. Change the following common fractions to decimals:

$\frac{1}{8}=?$	$\frac{3}{4}=?$	$\frac{4}{5}=?$	$\frac{3}{8}=?$
$\frac{5}{8}=?$	$\frac{2}{5}=?$	$\frac{7}{8}=?$	$\frac{3}{16}=?$
$\frac{7}{16}=?$	$\frac{3}{5}=?$	$\frac{5}{16}=?$	$\frac{5}{32}=?$

XCVI.

1. Change the following decimals to common fractions *by omitting the decimal point and supplying the proper denominator:*

$.5=\frac{5}{10}=\frac{1}{2}$	$.26=\frac{26}{100}=\frac{13}{50}$	$.125=\frac{125}{1000}=\frac{1}{8}$
$.6=\frac{6}{10}=\frac{3}{5}$	$.4=\frac{4}{10}=\frac{2}{5}$	$.8=\frac{8}{10}=\frac{4}{5}$
$.36=?$	$.48=?$	$.72=?$

2. $.625=?$ $.375=?$ $.875=?$
3. $.0625=?$ $.14=?$ $.00625=?$
4. Change $\frac{17}{32}$ to a decimal.
5. Change .75 to a common fraction.
6. Change $\frac{5}{50}$ to a decimal.
7. Reduce .84 to a common fraction.
8. Change $\frac{5}{64}$ to a decimal.
9. Change .45 to a common fraction.

XCVII.

1. $5\frac{1}{3} \div \frac{2}{3} = ?$ $5\frac{1}{4} \div \frac{3}{4} = ?$ $8\frac{3}{4} \div 2\frac{1}{4} = ?$
2. $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5} = ?$ $\frac{1}{8}$ of $\frac{16}{17} \times \frac{17}{2} = ?$ $\frac{5}{8}$ of $\frac{4}{10} = ?$
3. $\frac{3\frac{1}{3}}{4\frac{2}{3}}$ $\frac{8\frac{1}{10}}{\frac{3}{20}}$ $\frac{7\frac{1}{6}}{43}$
4. Change $\frac{17}{64}$ to a decimal.
5. Change .875 to common fraction.
6. Change $\frac{19}{32}$ to a decimal.
7. Reduce .1875 to a common fraction.
8. The following common fractions and their equivalent decimals are used so often that you must commit them to memory.

$\frac{1}{2}$=.5	$\frac{1}{4}$=.25	$\frac{3}{4}$=.75
$\frac{2}{5}$=.4	$\frac{3}{5}$=.6	$\frac{1}{8}$=.125
$\frac{3}{8}$=.375	$\frac{5}{8}$=.625	$\frac{7}{8}$=.875

Change these common fractions to decimals by the rule, and learn whether the correct decimal has been placed after each fraction.

(The attention of teachers is directed to the fact that every principle of the two years' work will now be brought under review in every six or seven lessons to the close of the book. In other words, the pupil will review the work about six times in the remaining lessons.)

XCVIII.

1. Reduce the following fractions to their lowest terms, also write under them how you do it. $\frac{24}{64}$, $\frac{15}{65}$, $\frac{27}{84}$, $\frac{48}{60}$.
2. Reduce to improper fractions $5\frac{5}{8}$, $7\frac{1}{8}$, $16\frac{2}{3}$, $75\frac{5}{6}$. Write how you do it.
3. Change these improper fractions to mixed numbers: $\frac{25}{9}$, $\frac{37}{12}$, $\frac{65}{18}$, $\frac{99}{24}$.
4. Change 8 to halves; 9 to thirds; 10 to fourths; 15 to fifths; 4 to tenths.

5. Change the following fractions to 48ths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{3}{24}$.

6. $\frac{5}{9}=\frac{}{18}$? $\frac{3}{17}=\frac{}{51}$? $\frac{7}{16}=\frac{}{64}$?

7. Change the following fractions to 36ths: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{18}$, $\frac{7}{9}$, $\frac{5}{12}$.

8. What will be the cost of $\frac{5}{9}$ of a hogshead of molasses at 10 cents a quart?

9. What will be the cost of $\frac{3}{8}$ of a gallon of wine at 30 cents a pint?

10. What will $\frac{5}{8}$ of a bushel of chestnuts cost at 15 cents a quart?

XCIX.

1. $\frac{1}{7}+\frac{5}{7}+\frac{3}{7}=$? $2\frac{1}{2}+\frac{5}{8}=$? $\frac{3}{5}+\frac{6}{7}+1\frac{1}{15}=$?

2. $\frac{1}{3}+\frac{1}{9}+\frac{5}{24}=$? $\frac{3}{7}+\frac{5}{3}+\frac{1}{2}=$? $6\frac{3}{8}+5\frac{4}{7}=$?

3. $\frac{7}{11}-\frac{3}{11}=$? $\frac{8}{17}-\frac{5}{17}=$? $\frac{5}{8}-\frac{1}{2}=$?

4. $\frac{7}{9}-\frac{2}{3}=$? $15\frac{5}{9}-4\frac{1}{3}=$? $7\frac{5}{7}-\frac{2}{3}=$?

5. $\frac{3}{4}$ of 48=? $\frac{5}{7}$ of 56=? $\frac{7}{10}$ of 90=?

6. Change the following fractions to units: $\frac{15}{7}$, $\frac{18}{9}$, $\frac{80}{9}$, $\frac{50}{11}$, $\frac{65}{12}$.

7. How many yards in $\frac{5}{8}$ of a yard? In $\frac{14}{7}$?

8. How many thirds in 12? In $8\frac{2}{3}$? In $12\frac{1}{3}$?

9. What is a prime number? Write the prime numbers to 97.

10. I paid \$24 for an overcoat, which was $\frac{3}{7}$ of all I had; how much money had I?

11. I bought 19 spoons, weighing 3 lbs., 2 oz., at \$1.50 an ounce. What did each spoon cost?

Ans. \$3.

C.

1. $\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}=$? $2\frac{1}{2}\times5\frac{1}{3}\times\frac{3}{16}=$?

2. $\frac{15}{7}\div3=$? $\frac{18}{4}\div\frac{3}{4}=$? $\frac{27}{5}\div\frac{3}{5}=$?

3. $\frac{24}{31}\div8=$? $\frac{18}{21}\div\frac{3}{21}=$? $\frac{10}{13}\div5=$?

4. $\frac{5}{6}\div\frac{2}{3}=$? $\frac{5}{8}\div\frac{5}{7}=$? $3\frac{2}{3}\div4=$?

5. Find the greatest common divisor of 60, 90, 150.

6. Find the least common multiple of 60, 90, 150.

7. In $\frac{1}{2}$ how many sixths? How many fourths? How many eighths? How many tenths? 12ths?

8. $8 \div \frac{2}{3} = ?$ $9 \div \frac{4}{5} = ?$ $7 \div \frac{5}{9} = ?$

9. $1 \div \frac{1}{2} = ?$ $1 \div \frac{1}{3} = ?$ $1 \div \frac{1}{7} = ?$

10. 2 times $8\frac{5}{6} = ?$ 5 times $3\frac{3}{4} = ?$ 9 times $7\frac{2}{7} = ?$

11. A has \$540, B has $\frac{1}{2}$ of A's and \$100 more, C has $\frac{3}{5}$ of A's and B's, D $\frac{1}{4}$ as much as A B and C; how much have all? *Ans.* 1820.

12. How many inches of surface in 6 one-inch cubes?

13. How many inches of surface in a six-inch cube? Make the cubes.

CI.

1. $\frac{1}{2}$ of $\frac{1}{3} = ?$ $\frac{1}{2}$ of $\frac{1}{4} = ?$ $\frac{1}{2}$ of $\frac{1}{5} = ?$
   $\frac{1}{3}$ of $\frac{1}{5} = ?$ $\frac{1}{4}$ of $\frac{1}{6} = ?$ $\frac{1}{5}$ of $\frac{1}{8} = ?$

2.

lbs.	oz.	pwt	gr.
37	10	15	15
35	11	14	16
18	9	10	10
25	2	15	17

	lbs.	oz.	pwt.	gr.
From	25	7	11	14
Take	18	8	17	15

3. Multiply 5 mi., 30 rds., 3 yds., 2 ft., 10 in. by 36.

4. Divide 1540 A., 140 sq. rds., into 20 lots.

5. 16 is $\frac{4}{5}$ of what number? 35 is $\frac{5}{7}$ of what number?

6. Change $\frac{3}{5}$ to tenths; to twentieths; to thirtieths.

7. How often is $\frac{1}{3}$ contained in 1? $\frac{1}{3}$ in 1? $\frac{3}{8}$ in 1?

8. A tailor had $67\frac{1}{4}$ yards of cloth in one piece; he sold at different times $10\frac{1}{2}$ yards, $12\frac{3}{8}$ yards, $15\frac{3}{8}$ yards: how much was left in the piece?

Ans. $28\frac{17}{24}$ yds.

9. How many yards in 1 mile?

10. How many cubic yards in 60000 cubic inches?

Ans. 1 cu. yd., 7 cu. ft., 1248 cu. in.

CII.

1. Write the decimal after each of these common fractions: $\frac{9}{10}$, $\frac{6}{10}$, $\frac{15}{100}$, $\frac{75}{100}$, $\frac{35}{1000}$, $\frac{1225}{10000}$, $\frac{5}{10000}$, $\frac{525}{10000}$, $\frac{7645}{10000}$.

2. Write the common fraction after each of these decimals: .5, .37, 1.25, .175, .028, .575, .1505, .00125, .08754, .18035, .000065, .140375.

3. Add 9 units and 39 thousandths; 208 units and 75 hundredths; 929 units and 4278 ten-thousandths; 35 units and 5 tenths; 64 units, 735 ten-thousandths, and 6 tenths.

4. Subtract .0875 from 2.2 Take 1 tenth from 1 unit.

5. $.87 \times .005 \times 8000 = ?$ $27.003 \times 5.007 = ?$

6. Write the rules for multiplication and division of decimals.

7. $.0125 \div 2.5 = ?$ $25 \div .0025 = ?$ $100 \div .01 = ?$

8. Change the following common fractions to decimals: $\frac{5}{8}$, $\frac{3}{5}$, $\frac{13}{16}$, $\frac{3}{32}$, $\frac{1}{12}$.

9. If 3.5 yards of silk cost $17.85, what will 9.6 yards cost?

(Let the pupil lay a narrow strip of paper over each decimal part, and then analyze the question.)

CIII.

1. Write from memory the complete Table of Principles.

2. Write from memory the diagram of fractions in Lesson LXXVI.

3. Change the following fractions to 64ths: $\frac{1}{8}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{16}$, $\frac{17}{32}$, $\frac{1}{2}$.

4. Change the following fractions to 105ths: $\frac{1}{5}$, $\frac{1}{7}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{8}{15}$, $\frac{16}{35}$.

5. $\frac{5}{16}-\frac{1}{16}=?$ $\frac{7}{8}-\frac{3}{5}=?$ $\frac{5}{16}-\frac{1}{7}=?$

$5\frac{1}{3}-4\frac{3}{4}=?$ $6\frac{7}{8}-4\frac{3}{4}=?$ $15\frac{1}{2}-12\frac{7}{10}=?$

6. What is cancellation? $\frac{5\times8\times19\times35}{25\times38\times24}$

7. $\frac{5}{8}\div5=?$ $5\div\frac{5}{8}=?$ $15\div\frac{3}{4}=?$ $\frac{5}{8}\div\frac{2}{3}=?$ $16\frac{1}{2}\div9=?$

8. Write the rule for division of common fractions.

9. A farmer sold 3 lambs weighing respectively $45\frac{3}{8}$, $36\frac{1}{2}$, and $29\frac{1}{4}$ pounds, at $4\frac{1}{8}$ cents a pound. What did he receive for them? *Ans.* $5.34\frac{1}{8}$.

CIV.

1. Place a fraction on the board, and by working with it answer the following questions:

What effect does multiplying both terms have?

What effect does dividing both terms have?

What effect does multiplying the numerator have?

What effect does dividing the numerator have?

2. What is a car load of 75 barrels of pork worth at $11\frac{3}{4}$ cents a pound? *Ans.* $1762.50.

3. What is a car load of 75 barrels of flour worth at $3\frac{3}{4}$ cents a pound? *Ans.* $551.25.

4. 6 times $\frac{3}{5}=?$ 7 times $\frac{5}{8}=?$ 8 times $\frac{5}{8}=?$

5. 3 times $2\frac{1}{2}=?$ 10 times $3\frac{3}{5}=?$ 12 times $4\frac{4}{5}=?$

6. When is a fraction at its lowest terms?

Reduce the following fractions to their lowest terms: $\frac{15}{35}$, $\frac{25}{40}$, $\frac{16}{}$, $\frac{56}{72}$, $\frac{144}{240}$.

7. How many acres in a field 1 mile long and 160 rods wide? *Ans.* 320 acres.

8. How many yards in $5\frac{3}{4}$ miles? *Ans.* 10120 yds.

CV.

1. Find the prime factors in 75, 120, 135.

$$\begin{array}{r|l} 5 & 75 \\ 3 & 15 \\ 5 & \end{array}$$

$5\times3\times5$ are the prime factors of 75.

$$\begin{array}{r|l} 2 & 120 \\ 3 & 60 \\ 2 & 20 \\ 2 & 10 \\ 5 & \end{array}$$

$2\times3\times2\times2\times5$ are the prime factors of 120.

$$\begin{array}{r|l} 5 & 135 \\ 3 & 27 \\ 3 & 9 \\ 3 & \end{array}$$

$5\times3\times3\times3$ are the prime factors of 135.

2. What is a greatest common divisor of several numbers?

3. Find the greatest common divisor of 75, 120, 135.

4. The least common multiple of several numbers is the product of all their different prime factors. Thus: 3×4 are the prime factors of 12.
3×5 are the prime factors of 15.
3×7 are the prime factors of 21.
The least common multiple of 12, 15, and 21 must have all the different factors of these numbers; otherwise it would not contain these numbers: $3\times4\times5\times7=420$ is the least common multiple of 12, 15, 21.

5. Find the least common multiple of 2, 3, 6; 3, 4, 6; 4, 8, 12; 6, 9, 12; 6, 8, 12; 75, 120, 135.

6. $37.15 \times 6 \times .44 \times 5000 = ?$

CVI.

1. Write the rules for the division of common fractions.

2. How often is $\frac{1}{2}$ contained in 2? In 3? In 4?

3. $9 \div \frac{1}{5} = ?$ $5 \div \frac{1}{8} = ?$ $9 \div \frac{1}{6} = ?$ $12 \div \frac{1}{12} = ?$ $10 \div \frac{1}{10} = ?$

4. $\frac{4}{5} = \frac{?}{25}$? $\frac{6}{8} = \frac{?}{48}$? $\frac{5}{9} = \frac{?}{81}$? $\frac{7}{10} = \frac{?}{70}$?

5. $\frac{3}{7} = \frac{?}{63}$? $2 = \frac{?}{3}$? $4 = \frac{?}{5}$? $8 = \frac{?}{8}$?

6. $9 = \frac{?}{8}$? $10 = \frac{?}{12}$? $12 = \frac{?}{4}$? $15 = \frac{?}{3}$?

7. $1 \div \frac{3}{4} = ?$ $1 \div \frac{5}{8} = ?$ $1 \div \frac{6}{7} = ?$ $1 \div \frac{5}{10} = ?$

8. $7 \times 9 \times 12 \times 15$ divided by $18 \times 5 \times 6$.

9. What number must be taken from 45 that the remainder may be $27\frac{5}{8}$?

10. What number multiplied by $5\frac{1}{2}$ will give a product of $16\frac{5}{8}$?

Can this question be worked by multiplication, when you already have the product?

CVII.

1. Add 57 and 45 hundredths; 39 and 7 tenths; 333 and 45 thousandths; 847 and 27 millionths; 5 and 1875 ten-thousandths.

2. Find the difference between 50, and 5 tenths; 60, and 60 hundredths; 70, and 7 ten-thousandths; 800, and 875 thousandths.

3. $75.36 \times .0875 = ?$ Prove.

4. $2.625 \div 35 = ?$ Prove.

5. $.00421155 \div .0573 = ?$ Prove.

6. $3073.146 \div 762 = ?$ Prove.

7. $89394 \div 3.804 = ?$ Prove.

8. A quantity of provisions will last 21 men 8 days; how long will it last 4 men?

9. If a man eat $1\frac{2}{3}$ pounds of bread in a day, how long will it take him to eat $16\frac{2}{3}$ pounds?

CVIII.

1. What is $\frac{1}{3}$ of 2? Of 4? Of 5? Of 7?
2. What is $\frac{1}{4}$ of 2? Of 5? Of 7? Of 9?
3. What is $\frac{1}{7}$ of 3? Of 6? Of 8? Of 11?
4. What is $\frac{1}{9}$ of 5? Of 17? Of 14? Of 25?
5. What are $\frac{2}{3}$ of 5? Of 6? Of 7? Of 9?
6. What are $\frac{5}{6}$ of 7? Of 9? Of 11? Of 13?
7. What are $\frac{3}{5}$ of 4? Of 9? Of 14?
8. How many inches in 1 gallon?
9. How many gallons will a box hold that is 1 yard long, 2 ft. wide, and 7 inches deep? *Ans.* $26\frac{2}{11}$ gal.
10. If a man's pay is $\$\frac{7}{8}$ a day, and a boy's pay is 50 cents a day; what will remain out of \$20 if you pay 3 men and 3 boys for 3 days' work? *Ans.* $\$7\frac{5}{8}$.

CIX.

1. $\frac{3}{64}$ $\quad 17\frac{5}{8}$ $\quad \frac{5}{8}$
2. $\frac{1}{3}+\frac{2}{3}+\frac{8}{3}=?$ $\quad \frac{1}{2}+\frac{1}{3}+\frac{1}{4}=?$ $\quad \frac{5}{8}-\frac{3}{8}=?$
3. $\frac{3}{4}$ of 64 $\quad \frac{3}{8}$ of $56=?$ $\quad 64$ is $\frac{8}{9}$ of what?
4. $\frac{1}{4}$ of $\frac{2}{3}=?$ $\quad \frac{3}{8}\times 5\frac{1}{2}\times\frac{4}{11}=8$ $\quad \frac{6}{7}\div 3=?$
5. $8\div\frac{4}{7}=?$ $\quad \frac{5}{9}\div 1\frac{1}{2}=?$
6. Find the greatest common divisor of 80, 160, 64.
7. Find the least common multiple of 80, 160, 64.
8. $15\frac{3}{8}-2\frac{2}{5}=?$ $\quad \frac{3\frac{1}{2}}{5\frac{1}{4}}$ $\quad \frac{8\frac{5}{8}}{7\frac{2}{3}}$
9. Add 62 and 25 hundredths; 175 and 404 thousandths; 145 and 7 ten-thousandths; 725 and 89 thousandths; 5065 millionths; 48 and 648 hundred-thousandths.

10. Take 25 tenths from 4 units; 135 hundredths from 75 units; 1875 thousandths from 30 units.

11. $76.048 \times .738 = ?$ Prove.

12. $\frac{17}{32}$ to a decimal.

13. .375 to a common fraction in its lowest terms.

CX.

1. What part of 1 pound is 1 ounce? 2 oz.? 3 oz.?

Ans. 1 ounce is $\frac{1}{16}$ of a pound. 2 ounces are $\frac{2}{16}$ or $\frac{1}{8}$ of a pound.

2. What part of 1 acre is 1 square rod? 5 sq. rds.? 30 sq. rds.? 120 sq. rds.? 40 sq. rds.?

3. What part of 1 ton is 1 cwt.? 5 cwt.? 9 cwt.?

4. What part of 1 cwt. is 1 pound? 20 lbs.? 25?

5. What part of 1 bushel is 1 peck? Is 1 quart? Is 1 pint? Is 5 quarts? Is 5 pints?

6. $\frac{1}{3}$ is equal to how many 6ths? 9ths? 12ths?

7. $\frac{2}{5}$ to 20ths. $\frac{5}{7}$ to 35ths. $\frac{5}{9}$ to 81sts. $\frac{5}{12}$ to 96ths. $\frac{7}{11}$ to 88ths. $\frac{5}{8}$ to 54ths.

8. $\frac{\frac{1}{2}}{\frac{1}{3}}$ $\frac{\frac{1}{4}}{\frac{5}{}}$ $\frac{\frac{1}{4}}{5\frac{1}{2}}$ $\frac{5}{\frac{1}{4}}$

9. Find the sum, difference, product, and quotient of $\frac{5}{8}$ and $\frac{3}{7}$.

CXI.

1. $\frac{25}{70}$ $\frac{45}{85}$ $\frac{63}{81}$ $\frac{91}{112}$ $\frac{108}{120}$

2. $2\frac{1}{2}$ $3\frac{1}{3}$ $4\frac{1}{4}$ $15\frac{1}{8}$ $27\frac{2}{7}$ $28\frac{3}{8}$

3. $\frac{13}{8}$ $\frac{15}{7}$ $\frac{47}{12}$ $\frac{79}{18}$ $\frac{97}{25}$ $\frac{274}{35}$

4. $\frac{1}{7}+\frac{5}{7}+\frac{9}{7}+\frac{11}{7}=?$ $\frac{1}{7}+\frac{1}{9}+\frac{1}{3}=?$

When the denominators are so large that you cannot tell what number to take for a common denominator, find the least common multiple of the denominators, and that will be the number for your common denominator. Thus: Add $\frac{7}{9}$, $\frac{5}{36}$, $\frac{8}{35}$, $\frac{5}{63}$. No one can tell what will

do for a denominator by looking at them. Place the denominators in a line and divide by any prime number that will divide two or more of the numbers; the quotients in the same way, until you cannot divide two of them. All the divisors and last quotients multiplied together will give the least common multiple.

$$\begin{array}{r|llll} 3) & 9 & 36 & 35 & 63 \\ \hline 3) & 3 & 12 & 35 & 21 \\ \hline 7) & 1 & 4 & 35 & 7 \\ \hline & 1 & 4 & 5 & 1 \end{array}$$

$3\times3\times7\times4\times5=1260$ is the L. C. M.

$\frac{7}{9}=\frac{980}{1260}$

$\frac{5}{36}=\frac{175}{1260}$

$\frac{8}{35}=\frac{288}{1260}$

$\frac{5}{63}=\frac{100}{1260}$

5. Add $\frac{5}{18}$, $\frac{7}{24}$, $\frac{12}{45}$.
6. Add $\frac{6}{28}$, $\frac{26}{60}$, $\frac{35}{90}$.
7. Add $\frac{24}{81}$, $\frac{35}{63}$, $\frac{97}{108}$.
8. $\frac{1}{2}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}=?$

$$)2 \quad 7 \quad 9 \quad 11$$

Placing the denominators in a line, you see there is no number that will divide two of the numbers. Then you must multiply them together to get the L. C. M. $2\times7\times9\times11=1386$ L. C. M.

CXII.

1. Find the prime factors in 48, 64, 80, 96.
2. Write the list of prime numbers up to 97.
3. What is a factor? What is a prime number?
4. $\frac{1}{2}$ of $\frac{2}{3}=?$ $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{8}=?$ $\frac{3}{16}\times\frac{15}{18}\times\frac{8}{30}=?$
5. 3 times $2\frac{1}{2}=?$ 8 times $5\frac{3}{5}=?$ 9 times $10\frac{1}{3}=?$
6. $\frac{15}{24}-\frac{3}{24}=?$ $\frac{5}{8}-\frac{1}{2}=?$ $\frac{7}{9}-\frac{1}{3}=?$
7. $5\frac{1}{2}-2\frac{3}{5}=?$ $6\frac{2}{3}-4\frac{1}{4}=?$ $228\frac{2}{3}-119\frac{7}{8}=?$

$5\frac{1}{2}=5\frac{5}{10}$
$2\frac{3}{5}=2\frac{6}{10}$

When you cannot take the lower numerator from the upper numerator, you can adopt a short method of add-

ing the terms of the upper fraction and then subtracting.

8. Find the prime factors in 50, 150, 200, 175.

9. Find the L. C. M. of 2, 4, 6, 8, 24, and 48.

CXIII.

1. Write the decimals after these fractions:

$\frac{5}{100}$=? $\frac{1535}{1000}$=? $\frac{375}{1000000}$=?

$\frac{375}{100}$=? $\frac{1875}{10000}$=? $\frac{4784}{10000}$=?

$\frac{48}{1000}$=? $\frac{1875}{100000}$=? $\frac{4785}{10000000}$=?

2. Add 3 and 9 tenths; 45 thousandths; 45 and 175 thousandths; 485 and 27 ten-thousandths; 64 and 64 millionths; 87564 hundred-thousandths.

3. Find the difference between 5 tenths and 50; 25 hundredths and 25; 879 and 978.

4. 32.064×.0375=? Prove.

5. 1047.42÷.0759=? Prove.

6. $\frac{2}{3}$ of \$360 is $\frac{3}{8}$ of my money; how much have I?

Ans. \$640.

7. Change to decimals $\frac{37}{64}$, $\frac{25}{32}$, $\frac{7}{16}$, $\frac{5}{8}$.

8. Change to common fractions .125, .480, .1875, .0875

9. I bought 5 pounds of coffee at \$$\frac{3}{8}$ a pound, 15 pounds of rice at \$$\frac{2}{7}$ a pound, 12 pounds of tea at \$$1\frac{1}{4}$ a pound. I paid for the purchase in wheat at \$$\frac{5}{8}$ a bushel; how many bushels did I pay?

Ans. 33$\frac{4}{7}$ bushels.

CXIV.

1. What part of a day is 1 hour? 2 hours? 8 hours? 10 hours? 12 hours? 18 hours?

2. What part of an hour is 1 minute? 10 minutes? 20 minutes? 30 minutes? 48 minutes?

3. $\frac{1}{3}$ of 4=? $\frac{1}{5}$ of 9=? $\frac{2}{3}$ of 14=? $\frac{5}{8}$ of 25=?

4. How many fourths in one-half? Tenths in one-

half? Ninths in one-third? Eighths in one-fourth? Twelfths in $\frac{1}{3}$? 24ths in $\frac{1}{8}$?

5. What is the weight of 9 bars of lead, if each bar weighs 7 lb., $11\frac{2}{3}$ oz.?

6. A goes at the rate of $8\frac{4}{5}$ ft. a second, B at the rate of $8\frac{5}{9}$ ft. a second; which goes farther in 5 minutes, and how much? *Ans.* $73\frac{1}{3}$ ft.

7. If 16 pounds of butter are worth $4.80, what are 14.5 pounds worth?

8. If 5 horses eat 37.5 bushels of oats in 4 days, how much will 10 horses eat in half that time?

Ans. 37.5 bushels.

9. Find the L. C. M. of 2, 4, 8, 16, 32, 64, 144.

Ans. L. C. M. 576.

CXV.

1. In 12875 square inches how many square yards?

2. What number multiplied by $4\frac{3}{4}$ will equal 9?

3. At $7\frac{1}{3}$ cents a pound, how much sugar can be sold for 1 dollar?

4. A barrel holds $47\frac{1}{4}$ gallons, two kegs holding $12\frac{2}{3}$ gallons and $15\frac{5}{6}$ gallons are filled from it; how much remains in the barrel? *Ans.* $18\frac{3}{4}$ gallons.

5. Multiply the sum of $2\frac{7}{16}$ and $1\frac{5}{8}$ by their difference. Work the example by both common fractions and decimals.

6. What will 17 bu., 2 pk. of potatoes cost at $1.50 per bushel? Change 2 pecks to a decimal of a bushel.

7. These are the decimals that you must remember without having to work them every time you need them.

$\frac{1}{2}$=.5	$\frac{1}{4}$=.25	$\frac{3}{4}$=.75	$\frac{1}{8}$=.125
$\frac{3}{8}$=.375	$\frac{5}{8}$=.625	$\frac{7}{8}$=.875	$\frac{1}{16}$=.0625
$\frac{3}{16}$=.1875	$\frac{5}{16}$=.3125	$\frac{7}{16}$=.4375	

CXVI.

The pupil will notice the likeness in the following words:

DIVISOR.	MULTIPLE.
DIVISION.	MULTIPLICATION.

Now copy the divisors and some of the multiples of 20, 25, 30, 36, 48, as below on the board.

Divisors.		Multiples.
2, 4, 5, 10	20	20, 40, 60, 80, etc.
5, 5	25	25, 50, 75, 100, etc.
2, 3, 5, 6, 10, 15	30	30, 60, 90, 120, etc.
2, 3, 4, 6, 9, 12, 18	36	36, 72, 108, 144, etc.
2, 3, 4, 6, 12, 16, 24	48	48, 96, 144, 192, etc.

(The teacher will call for the divisors and about ten multiples to each number, the greatest common divisor and the least common multiple of any two or three of the numbers, and have them marked in heavier chalk on the board.)

1. Write from memory the diagram of fractions in Lesson LXXVI.

2. Write from memory the complete Table of Principles.

3. What is a complex fraction? Make five complex fractions and work each one.

4. Write rules for division of common fractions.

5. Write the rules for multiplication and division of decimals.

6. Reduce these fractions to decimals and find their sum: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{17}{20}$, $\frac{13}{16}$, $\frac{37}{50}$, and $\frac{11}{40}$. *Ans.* 6 0025

7. A field is 4 miles square. If you step a yard every second, how long will it take you to walk round the field? *Ans.* 7 hrs, 49 min., 20 sec.

8. If you step a yard every second, how far can you walk in 4 hrs. and 24 min.?

CXVII.

1. What is the difference between 5 sq. inches and five inches square?

2. What is the difference between 5 square inches and 5 cubic inches? Be careful.

3. How many cubic feet in a cellar 30 feet long and 20 feet wide? Think carefully.

4. Suppose a lawn 130 feet square, with a gate in the middle of each side: how many bricks (4x8 in.) are required to lay a pavement 6 feet wide from north to south, and from east to west through the ground?

Ans. 6858 bricks.

Make a drawing of the ground.

5. How many square feet in a path 30 yards long?

6. Divide 183 days, 18 hrs., 20 min., 12 sec. by 13. Prove.

7. How much cider did a farmer put in five dozen bottles, each holding 1 pint, 3 gills?

8. One barrel of flour was divided among 14 persons; how much did each one receive?

CXVIII.

1. What effect does placing a cipher on the right of .5 have. *Ans.* It changes it to .50, and $\frac{50}{100}=\frac{5}{10}$. You have simply multiplied both terms by 10. *You have changed its form without altering its value.*

2. Place a cipher on the right of .25 and explain.

3. Place a cipher on the right of .125 and explain.

4. $13\div\frac{6}{7}=?$ $\frac{1}{3}\div\frac{1}{2}=?$ $5\frac{3}{5}\div 2=?$

5. $\dfrac{\frac{1}{8}}{\frac{1}{4}}$ $\dfrac{7\frac{1}{2}}{3\frac{1}{3}}$ $\dfrac{3\frac{1}{5}}{9}$

6. $12.6\div.012=?$ $2.709\div 1.29=?$ $102048\div 3189=?$

7. $72.36\div.0036=?$ $25\div.0025=?$ $6250\div.25=?$

Prove these six examples.

CXIX.

1. What effect does placing a cipher on the right of .3 have? Explain.

2. What effect does placing a cipher on the right of .75 have? Explain.

3. Reduce these fractions:

$\frac{25}{65}$	$\frac{25}{8}$	$25\frac{3}{4}$	$\frac{2}{3}$
$\frac{36}{48}$	$\frac{67}{12}$	$67\frac{5}{8}$	$\frac{4}{5}$
$\frac{49}{63}$	$\frac{95}{18}$	$138\frac{3}{7}$	$\frac{5}{9}$

4. What are the terms of a fraction? and what do they show?

5. $\frac{18}{37}-\frac{5}{37}=?$ $\frac{29}{38}-\frac{17}{38}=?$ $2-\frac{7}{9}=?$

6. $3-2\frac{2}{7}=?$ $8-5\frac{5}{9}=?$ $12-6\frac{4}{5}=?$

7. $18\frac{2}{5}-13=?$ $25\frac{4}{7}+19=?$ $37-18\frac{3}{4}=?$

8. $\frac{2}{3}\times 1=?$ $\frac{3}{4}\times 0=?$ $\frac{5}{6}\times 2=?$

9. $\frac{5}{8}\times 3=?$ $1\times\frac{5}{8}=?$ $2\times\frac{4}{7}=?$

10. $\frac{3}{7}\times 5=?$ $4\times\frac{5}{7}=?$ $10\times\frac{3}{5}=?$

11. What is cancellation? Use cancellation in this example: $\frac{3}{5}\times\frac{15}{18}\times\frac{27}{63}\times\frac{2}{9}\times\frac{21}{54}=?$

CXX.

1. What part of \$5 is \$3? Is \$4? Is \$2?

2. What part of 1 mile is 1 rod? 1 yard?

3. What part of 1 mile is 30 rods? 80 rods?

4. What part of 1 mile is 176 yards? 440 yards?

5. What part of 1 week is 1 day? 3 days? 5 days?

6. Reduce 17650 square inches to higher denominations.

7. Reduce 27650 cubic inches to cubic yards.

8. Change 40655 pounds to tons.

9. Reduce 480 pints to bushels.

10. How many inches in 3 miles?

11. How many pints in 5 hogsheads of wine, and

how many bottles holding $1\frac{1}{2}$ pints may be filled from them? *Ans.* 1680 bottles.

CXXI.

1. How many seconds in 5 years?
2. Change 2246441 seconds to higher denominations.
3. How many dozen in 144 scores?

Ans. 240 dozen.

4. If 48 sheets of paper are required for 1 book, how many books may be made from 10 reams?
5. 5 loads of wheat weighed respectively 1860 lbs., 2300 lbs., 2000 lbs., 1760 lbs., 2400 lbs.; what was the wheat worth at $1.50 per bushel? *Ans.* $258.
6. Write 20 dollars and 50 cents; 37 cents, 5 mills; 10 dollars and eight mills; 175 dollars and 5 dimes; 68 dollars and 16 cents; $87\frac{1}{2}$ cents. Add. Prove.
7. How much silver in 6 dozen spoons, if one spoon weighs 15 pwt., 14 grains?

CXXII.

1. A grocer buys 30 dozen eggs at the rate of 5 dozen for 50 cents; what is the cost?
2. If 12 yards of cloth are worth $15, what are $7\frac{1}{2}$ yards worth? *Ans.* $9\frac{3}{8}$.
3. How many pounds of sugar can be bought for $8 at 8 cents a pound?
4. What is the distance round a field 800 rods long and 600 rods wide?
5. How many inches in $\frac{1}{2}$ a yard?
6. How many rods in a quarter of a mile?
7. What part of 1 yard is 12 inches? 18 in?
8. I sold 5 bu. oats at 15 cents a quart; what did I receive?

9. What part of a gallon is 1 quart? 1 pint? 1 gill?

10. Was the year 1860 leap year? Was 1866?

11. How many years in 10 centuries? What century was 1800?

12. In how many years may a man save $1920, saveing $8 a month? *Ans.* 20 years.

CXXIII.

1. In 2000 mills how many cents? Dollars?

2. How many cents in $4\frac{1}{2}$ eagles?

3. I paid $8 for 5 reams of paper; for how much must I sell it a quire to gain $4? *Ans.* 12 cents.

4. How many buttons in $\frac{1}{2}$ a gross?

5. At 30 cents a quire, what will $\frac{1}{4}$ of a ream of of writing paper cost?

6. How many cubic feet in a piece of timber 1 ft. square at each end and 8 feet long?

(Shape a stick 1 in. square, 8 in. long.)

7. How many cubic feet in a log 2 ft. square at each end and 8 ft. long? Shape a stick 2 in. square.

8. How many cubic inches in a 2-inch cube?

(Show the cubes.)

9. How many two-inch cubes in a 6-inch cube?

10. If an acre of land is 20 rods long, how wide is it? Draw the figure.

11. If an acre of land is 10 rods wide, how long is it?

CXXIV.

1. How many square yards in a floor 15 yards long and 12 feet wide?

2. How many square yards in a floor 15 ft., 9 in. long and 12 ft. wide?

3. What part of a foot is 3 inches? 6? 8?

4. Find the cost of 2 T., 8 cwt. of wheat at $1.50 a bushel? *Ans.* $120.

5. A grocer paid $6.50 for a barrel of flour, he sold it in 14-pound sacks at 75 cents a sack; did he gain anything?

6. The month of October came in on Tuesday; what was a man's pay who worked every day except Sunday at $2.50 a day?

7. I bought 40 gallons of sirup at 80 cents a gallon, sold it at 25 cents a quart; what was my profit? *Ans.* $8.

8. How many cubic inches in a forty-gallon barrel?

9. The sum of three numbers is 15, one of them is $2\frac{3}{4}$, another is $3\frac{5}{8}$; what is the other?

CXXV.

1. $42\frac{3}{4}$ gallons leaked from a hogshead of molasses; what was the remainder worth at $1\frac{1}{4}$ a gallon?

2. How often will a wheel 15 feet, 6 inches in circumference turn round in going 20 miles?

3. If 15 cows are worth $750, what is the value of 22 cows?

4. If 15.75 yards of cloth are worth $51.1875, what are 22.5 yards worth? Cover the decimals with small strips of paper and analyze before working.

5. If $7\frac{3}{4}$ cords of wood cost $35\frac{5}{8}$, what will $5\frac{2}{12}$ cords cost? Cover the fractions with strips of paper and analyze.

6. $3\frac{2}{3} \div 4\frac{4}{10} \times 8\frac{8}{10} = ?$ Work such examples in this way $\frac{11}{3} \times \frac{11}{4} \times \frac{88}{10} = ?$ *Ans.* $7\frac{1}{3}$.

(The teacher will show the class how the result is obtained by cancellation.)

CXXVI.

1. If $12.50 will buy a barrel of pork, what is a pound worth?

2. What is a denominate number?

3. What is a compound number?

4. My house is worth $4000, which is $\frac{2}{3}$ of the value of my store; what is the store worth?

5. My store is worth $4000, my goods $\frac{6}{8}$ of the value of the store; what is the value of my goods?

6. If 3 be subtracted from both terms of the fraction $\frac{5}{8}$, will the resulting fraction be greater or less than $\frac{5}{8}$?

7. If 4 be added to both terms of $\frac{8}{9}$, will the resulting fraction be greater or less than $\frac{8}{9}$?

8. If a pole is $\frac{2}{3}$ in the mud and $\frac{1}{6}$ in the water, how much is out of water?

The pole is a unit. $\frac{5}{6}$ from a unit will give you the length in the air.

(Bring the pole before the class.)

9. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{11}{6}$ divided by $\frac{1}{3}$ of $\frac{5}{9}$ of $\frac{18}{25}$=?

Dividend. Divisor.

$\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{11}{6} \div \frac{1}{3}$ of $\frac{5}{9}$ of $\frac{18}{25}$=?

$\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{11}{6} \times \frac{3}{1} \times \frac{9}{5} \times \frac{25}{18}$=? Cancel.

CXXVII.

1. If you multiply both terms of $\frac{1}{2}$ by 2, you get $\frac{2}{4}=\frac{1}{2}$. If you multiply both terms of $\frac{1}{2}$ by 3, you get $\frac{3}{6}=\frac{1}{2}$. If you multiply both terms of $\frac{1}{2}$ by 4, you get $\frac{4}{8}=\frac{1}{2}$. *You change the form, but do not alter the value.*

2. Place a cipher on the right of the decimal .9, and you have $.90=\frac{90}{100}$. What have you done? Simply multiplied both terms by 10.

3. .70 Erase the cipher on the right of .70, and

you get .7 What have you done? Simply divided both terms by 10.

4. Find the G. C. D. of 312 and 429. Of 630 and 595. Of 48, 63, and 111. The G. C. D. cannot be larger than the smallest number.

5. Find L. C. M. of 46, 69, and 253. Of 40, 60, 72, and 120. Of 24, 44, 65, and 100. The L. C. M. cannot be smaller than the largest number.

6. The least common multiple of several numbers is the *least number they will divide.*

CXXVIII.

1. These are the decimals you must remember:

$\frac{1}{2}$=.5 $\frac{1}{4}$=.25 $\frac{3}{4}$=.75 $\frac{1}{8}$=.125
$\frac{3}{8}$=.375 $\frac{5}{8}$=.625 $\frac{7}{8}$=.875 $\frac{1}{16}$=.0625
$\frac{3}{16}$=.1875 $\frac{5}{16}$=.3125

2. Reduce the following fractions to decimals, and add them: $\frac{6}{15}$, $\frac{1}{12}$, $\frac{13}{14}$.

$\frac{6}{15}$)6.0(.4 $\frac{1}{12}$)1.0000 $\frac{13}{14}$)13.0000(.9285
60 .0833$\frac{1}{3}$ 126
40
.4 28
.0833$\frac{1}{3}$ 120
.9285$\frac{5}{7}$ 112
80
70
$\frac{10}{14}$=$\frac{5}{7}$

The pupil will observe that where the fractions do not make even decimals, the decimals must be kept at the same number of places, so that the fraction remainders may fall under each other in the addition.

3. Reduce $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{9}$ to decimals, and add.

4. Change the following to common fractions and to their lowest terms:

.25	.5625	.53125	.375
.9375	.875	.3275	

5. A field contains an acre, its width is 7.25 rods; what is its length, to not more than four decimal places?

CXXIX.

1. The product of two numbers is 6.23; one of the numbers is 124.6, what is the other? To work such examples mark them thus:

Product.	One Number.	
6.23	124.6	124.6)6.230(.05
.05 is the other number.		6230

Proof 124.6 One number.
.05 Other number.
6.230 Product.

2. Write the rules for multiplication and division of decimals.

3. How many pounds of paper will \$17.2125 buy, at \$.27 a pound? Prove.

4. If \$42.57 buy 12.375 gallons of wine, what is a gallon worth?

5. Find the G. C. D. of 3285, 9125, 1095.

6. A man owned $\frac{3}{4}$ of a ship, and sold $\frac{3}{8}$ of his part; what part of the ship had he left?

Owned.	Sold.
$\frac{3}{4}$	$\frac{3}{8}$ of $\frac{3}{4}=\frac{9}{32}$
Sold $\frac{9}{32}$	
Had left	

7. Find the product of 107 ten-thousandths and 206 hundred thousandths, and to that add 1 ten-millionth.

CXXX.

1. A man has $\frac{2}{3}$ interest in a store; he dies leaving $\frac{1}{7}$ and $\frac{1}{9}$ to his son and daughter, the remaining part to his widow; what was the part the widow got?

2. What part of a pound is .2 ounces? 4 ounces? 6 ounces? 8 ounces?

3. What is the value of 3 A., 30 P. of land at $60.75 an acre?

As the price is by the acre, what are you going to do with the 30 poles? Change it to a decimal of an acre. There are 160 poles in an acre. 1 pole is $\frac{1}{160}$ of an acre, 30 poles $\frac{30}{160}$, or $\frac{3}{16}$ of an acre.

$\frac{3}{16})3.0000$ (.1875

160 Poles.)30.0000 Poles.(.1875

3 A., 30 P.=3.1875 A., at $60.75 an acre.

4. 4 bu., 2 pks. of chestnuts at $4.50 per bu.?

5. 5 gal., 1 qt. of oil at $1.25 a gallon?

CXXXI.

1. $\frac{64}{144}$
2. $48\frac{5}{16}$
3. $\frac{375}{28}$

$\frac{375}{625}$

$369\frac{11}{12}$

$\frac{655}{37}$

4. Change the following fractions to 72ds: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{15}{24}$, $\frac{29}{36}$.
5. $\frac{1}{8}+\frac{5}{9}+\frac{3}{16}+\frac{17}{18}=?$ $5\frac{1}{2}+5\frac{5}{6}+7\frac{3}{7}=?$
6. $9\frac{3}{8}-7\frac{3}{16}=?$ $242\frac{5}{32}-179\frac{1}{3}=?$
7. $\frac{5}{9}\times 9=?$ $18\times\frac{3}{5}=?$ $\frac{15}{16}\times\frac{2}{3}\times\frac{5}{6}=?$
8. $\frac{5}{19}\div\frac{2}{3}=?$ $9\div\frac{3}{8}=?$ $\frac{5}{8}\div 10=?$
9. $\frac{4\frac{1}{2}}{5\frac{1}{2}}$ $\frac{6\frac{3}{7}}{8\frac{8}{9}}$
10. 160 is $\frac{5}{9}$ of what number?
11. 800 is $\frac{4}{15}$ of what number?

12. How many eighths in $\frac{1}{2}$? How many 9ths in $\frac{1}{3}$? How many 16ths in $\frac{1}{4}$?

13. 5 times $\frac{3}{8}$ = ? 9 times $\frac{4}{7}$ = ?

14. 5 times $3\frac{1}{3}$ = ? 9 times $4\frac{4}{7}$ = ?

CXXXII.

1. Add 25 tenths; 375 hundredths; 45 ten-thousandths; 487 millionths; 44 units and 87 hundred-thousandths.

2. Find the difference between 82 hundredths and 82; between 4.5 and 6 units 5 hundredths; between 10 and 1 tenth.

82.375 − 75.5 = ? 74.125 − 28.597 = ?

3. 71.009 × 2.5 = ? 37.07 × 64.046 = ?

4. 900 ÷ .375 = ? 100 ÷ .125 = ?

5. 10 ÷ .25 = ? 1 ÷ .8 = ?

6. 25 ÷ .005 = ? 147 ÷ 2.1 = ?

7. If $\frac{3}{5}$ of a bushel of chestnuts are worth 60 cents, what are $\frac{3}{4}$ of a bushel worth?

Notice you must find the value of 1 bushel before you can find that of $\frac{3}{4}$ of a bushel; and notice further, that when you want to find the value of 1 bushel, or 1 yard, or 1 pound, etc., you must divide. $60 \div \frac{3}{5} = 100$ cents, the price of a bushel. You may get it by analysis also.

$\frac{3}{5}$ cost 60 cents.

$\frac{1}{5}$ costs 20 cents.

$\frac{5}{5}$, or a bushel, cost 100 cents.

CXXXIII.

1. What are $\frac{3}{5}$ of a ton of iron worth at $\$18\frac{3}{4}$ a ton?

2. If $3\frac{4}{7}$ yards of cloth cost $\$14\frac{5}{9}$, what is 1 yard worth?

3. A number plus its half equals 12; what is the number?

Do you not see we must call the number $\frac{2}{2}$, and

$\frac{2}{2}+\frac{1}{2}=\frac{3}{2}=12$.

If $\frac{3}{2}=12$

$\frac{1}{2}=4$

$\frac{2}{2}=8$. 8 is the number.

4. A number plus its $\frac{1}{3}$ equals 12; what is the number?

Do you not see we must call the number $\frac{3}{3}$, and

$\frac{3}{3}+\frac{1}{3}=\frac{4}{3}=12$.

5. What number plus its $\frac{2}{6}$ equals 84?

6. What number plus its $\frac{5}{8}$ equals 26?

7. What number plus its $\frac{1}{2}$ and $\frac{1}{3}$ equals 44?

We look at the denominators, and determine that we must call the number $\frac{6}{6}$.

8. If $\frac{5}{8}$ of a farm are worth \$600, what are $\frac{7}{8}$ of it worth?

9. Add 37 tenths; 138 hundredths; $4\frac{1}{5}$; 175 thousandths; $10\frac{1}{8}$; 8.003; $\frac{1}{3}$; $27.916\frac{1}{3}$

CXXXIV.

1. If $46\frac{3}{8}$ bushels of clover seed cost \$$174\frac{3}{4}$, what will $66\frac{2}{3}$ bushels cost? *Ans.* \$250.

2. If $\frac{1}{16}$ of my money is \$660, what is $\frac{5}{8}$ of it?

3. Multiply $3.2\times\frac{1}{2}\times200\times\frac{1}{8}\times\frac{175}{1000}\times10=$?

4. Reduce $\frac{2}{3}$, $\frac{3}{7}$, $\frac{5}{9}$ to decimals and add.

5. Reduce $\frac{17}{20}$, $\frac{13}{16}$, $\frac{37}{50}$, and $\frac{11}{40}$ to decimals and add.

6. What part of a pound is 5 ounces? 12 ounces?

7. What part of a day is 3 hours? 20 hours?

8. What part of a cord is 16 cubic feet?

9. What part of a bushel is 2 quarts? 3 pints?

10. Tell the difference between 7 square inches and 7 inches square. Use the scissors.

CXXXV.

1. Which would you rather have, $\frac{7}{8}$ of \$5600 or $\frac{4}{7}$ of it?

2. \$5600 is $\frac{7}{8}$ of what number?

3. Tell the difference between 14 square inches and a 7-inch cube.

4. How long is an acre? How wide?

5. A field has the width of a mile and the length of 5 miles, how many yards of fence will it require to inclose the whole and to lay it off into fields each 1 mile square? *Ans.* 28160 yards.

6. How many *cubic* feet in a block of marble 4 ft. *long* and 3 feet *wide?*

7. A day's work is 10 hours; what is my pay for 10 days, 5 hours' work at \$2.50 a day?

As the pay is by the day, what will you do with the 5 hours?

8. What effect does placing two ciphers on the right of .8 have? Explain.

9. Give the value of 10 A., 16 P. of land at \$40 an acre.

As the price is for an acre, what are you going to do with the 16 P.?

CXXXVI.

1. If I erase the cipher on the right of .50, what have I done?

I have changed .50 to .5 or $\frac{5}{10}$ and $\frac{50}{100}=\frac{5}{10}$; I have simply divided both terms of $.50=\frac{50}{100}$ by 10.

2. What effect is had by erasing the two ciphers on the right of .500?

3. What is a prime factor of a number?

4. Write the prime numbers up to 97.

5. Find the G. C. D. of 120, 168, 312, 480.

The G. C. D. can have no other prime factors than those that are common to all the numbers.

2)120	2)168	2)312	2)480
2)60	2)84	2)156	2)240
2)30	2)42	2)78	2)120
3)15	3)21	3)39	2)60
5	7	13	2)30
			3)15
			5

Examine the above factors; you see each number has three 2s and one 3.

$2\times2\times2\times3=24$ is the G. C. D.

6. Find G. C. D. of 96, 144, 129; 370, 555, 1850; 154, 484, 2200.

7. A least common multiple of several numbers must have all the different prime factors as often as they appear in each number.

Find the L. C. M. of 21, 80, 120.

3)21	80	120	
2) 7	80	40	
2) 7	40	20	
2) 7	20	10	
5) 7	10	5	
7	2	1	L. C. M. $3\times2\times2\times2\times5\times7\times2=1680$

8. Find the L. C. M. of 24, 60, 72, 100.

9. Find the L. C. M. of 2, 3, 4, 5, 6, 7, 8, 9, 10.

10. Find the G. C. D. and L. C. M. of 4, 8, 12.

11. Is the L. C. M. of several numbers always greater than the G. C. D. of the same numbers?

CXXXVII.

1. $.3 \times .05 = .015 = \frac{3}{10} \times \frac{5}{100} = .015$
2. $.15 \times .15 = .0225 = \frac{15}{100} \times \frac{15}{100} = \frac{225}{10000} = .0225$
3. $.75 \div .5 = 1.5 = \frac{75}{100} \div \frac{5}{10} = \frac{75}{100} \times \frac{10}{5} = \frac{15}{10} = 1\frac{5}{10}$
4. $.64 \div .08 = 8 = \frac{64}{100} \div \frac{8}{100} = 8$
5. $.125 \div .25 = .5 = \frac{125}{1000} \div \frac{25}{100} = \frac{125}{1000} \times \frac{100}{25} = \frac{1}{2} = \frac{5}{10}$
6. $.065 \times 07 = .00455 = \frac{65}{1000} \times \frac{7}{100} = \frac{455}{100000} = .00455$

Work above examples until you can work them without the book.

7. Add

	sq. yds.	sq. ft.	sq. in.
	6	7	87
		4	120
	3	2	7
	9	1	10
	7	2	104

	sq. yds.	sq. ft.	sq in.
From	9	1	10
Take	7	2	104

8.

	wks.	da.	hrs.	min.	sec.
From	8	3	16	20	14
Take	7	4	11	37	19

	gal.	qts.	pt.	gl.
	15	3	1	3
				24

9. 186 sq. yds., 5 sq. ft., 50 sq. in. divided by 13.
10. $.075 \times .025 = .001875$ $\frac{75}{1000} \times \frac{25}{1000} = \frac{1875}{1000000}$.
11. $1000 \div .125 = ?$ $100 \div .125 = ?$

CXXXVIII.

1. Write the complete Table of Principles.
2. Write the diagram of fractions in Lesson LXXVI.
3. Find the prime factors in 378 and 490. In 480 and 240.
4. Write the two rules for division of common fractions.

Work this example both ways: $\frac{5}{8} \div \frac{5}{16} = ?$

$\frac{5}{8} \div \frac{5}{16} = \frac{10}{16} \div \frac{5}{16} = \frac{10}{5} = 2$ $\frac{5}{8} \div \frac{5}{16} = \frac{5}{8} \times \frac{16}{5} = \frac{80}{40} = 2$

5. How many thirds in 4? In 6? In 8? In 12?

6. How many fifths in 6? In 8? In 12?

7. How many decimal places are required to write tenths? Thousandths? Hundredths? Millionths? Ten-thousandths? Hundred-thousandths?

8. What effect is had by placing two ciphers on the right of .45?

CXXXIX.

1. What is $\frac{1}{3}$ of 2? $\frac{1}{3}$ of 3? $\frac{1}{3}$ of 4? $\frac{1}{3}$ of 7?
2. What is $\frac{1}{4}$ of 5? $\frac{1}{4}$ of 7? $\frac{1}{4}$ of 9? $\frac{1}{4}$ of 11?
3. What is $\frac{1}{5}$ of 6? $\frac{1}{5}$ of 7? $\frac{1}{5}$ of 8? $\frac{1}{5}$ of 9?
4. How many 1s in $1\frac{3}{4}$? In $1\frac{5}{8}$? In $2\frac{5}{7}$?
5. $\frac{1}{2}$ of $\frac{1}{2}$=? $\frac{1}{2}$ of $\frac{1}{3}$=? $\frac{1}{2}$ of $\frac{1}{4}$=? $\frac{1}{2}$ of $\frac{1}{5}$=?
6. $\frac{1}{3}$ of $\frac{1}{3}$=? $\frac{1}{3}$ of $\frac{1}{4}$=? $\frac{1}{3}$ of $\frac{1}{5}$=? $\frac{1}{3}$ of $\frac{1}{6}$=?
7. $\frac{2}{3}$ of $\frac{3}{4}$=? $\frac{3}{4}$ of $\frac{4}{5}$=? $\frac{5}{6}$ of $\frac{7}{8}$=? $\frac{7}{8}$ of $\frac{1}{10}$=?
8. $\frac{2}{3}$ of 3=? $\frac{3}{5}$ of 4=? $\frac{5}{6}$ of 7=? $\frac{7}{8}$ of 1=?
9. How many inches in 1 gallon?
10. How many gallons can be contained in a box that is 22 ft. long, 14 ft. wide, and 9 feet deep?

Ans. 20736 gallons.

11.

.5	.24	.36	.036
2.4	2.24	3.36	.436
.1875	.3125	.4375	.35725
.15625	.78125	.390625	

Write the corresponding common fractions to these decimals and reduce to lowest terms.

CXL.

1.

$\frac{35}{10}$	$\frac{9}{10}$	$\frac{25}{100}$	$\frac{75}{100}$
$\frac{175}{100}$	$\frac{144}{1000}$	$\frac{386}{1000}$	$\frac{1386}{1000}$
$\frac{1800}{10000}$	$\frac{1875}{10000}$	$\frac{156}{1000000}$	$\frac{3156}{1000000}$
$\frac{13156}{1000000}$	$\frac{1890}{1000000}$	$\frac{57658}{1000000}$	$\frac{3\frac{1}{2}}{100}$

Write the corresponding decimals.

2. Find the sum, product, difference, and quotient of .0625 and .25

3. Add 45 hundredths; 345 tenths; 62 thousandths; $3\frac{1}{4}$; 7 and 7 millionths; $52\frac{1}{8}$; $\frac{253}{1000}$; $\frac{253}{100000}$.

4. If 3 be added to both terms of $\frac{7}{9}$, will the resulting fraction be larger or smaller than $\frac{7}{9}$?

5. Will the fraction be increased or diminished if you take 3 from both terms of $\frac{7}{9}$?

Taking 3 from both terms gives $\frac{4}{6}$. Is $\frac{4}{6}$ greater or less than $\frac{7}{9}$?

6. This question is for the smart boys and girls. If 5 be taken from both terms of $\frac{6\frac{1}{2}}{7\frac{1}{2}}$ will the resulting fraction be greater or less than $\frac{6\frac{1}{2}}{7\frac{1}{2}}$?

7. When you wish to get the value of 1 bushel, or 1 yard, 1 pound, etc., you must divide.

8. If $\frac{5}{9}$ of a farm cost \$750, what will $\frac{8}{9}$ of it cost?

Ans. \$810.

CXLI.

1. Which is greater, $\frac{2}{3}$ or $\frac{5}{8}$? and how much?

2. The difference between $\frac{2}{3}$ and $\frac{5}{8}$ of my money is \$300; how much have I? *Ans.* \$7200.

3. What is $\frac{3}{4}$ of \$880? $\frac{7}{10}$ of \$880?

4. The difference between $\frac{3}{4}$ of my money and $\frac{7}{10}$ of my money is \$44; how much have I?

5. Find the difference between 5 tenths and 75 hundredths; between 1875 thousandths and 20 units.

6. Find the difference between $2\frac{1}{2}$ and .75; between $3\frac{1}{5}$ and $2\frac{1}{3}$; between $7\frac{1}{7}$ and $6\frac{5}{9}$.

7. Find the difference between 48 units and 48 tenths; between $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ and $\frac{5}{8}$ of $\frac{4}{7}$ of 3.

8. $5-4\frac{1}{3}=?$ $7-3\frac{3}{5}=?$ $18-14\frac{4}{7}=?$ $19-\frac{1}{2}=?$

9. Multiply .75 by $\frac{2}{3}$; .175 by $\frac{3}{4}$.

10. If $\frac{8}{9}$ of a barrel of flour is worth \$5.60, what would $\frac{3}{7}$ be worth?

11. Write rules for multiplication and division of decimals.

CXLII.

1. If I have a pie and give $\frac{1}{2}$ of it, I have what part left? If I have a pie and give $\frac{1}{3}$ of it, what part have I left?

2. If I have $\frac{3}{4}$ of a pie and give $\frac{1}{2}$ of it, do I have $\frac{1}{2}$ a pie left?

3. If I have $\frac{3}{4}$ of a pie and give $\frac{1}{3}$ of it, what part have I left? Why $\frac{2}{3}$ of it? Is it $\frac{2}{3}$ of the pie, or $\frac{2}{3}$ of $\frac{3}{4}$ of the pie?

4. If I have money and spend $\frac{1}{4}$ of it, I have left $\frac{3}{4}$ of it.

5. If I spend $\frac{1}{3}$ of my money and have \$20 left, how much had I at first?

6. If I spend $\frac{2}{5}$ of my money and have \$30 left, what had I at first?

7. If I own $\frac{3}{4}$ of a house and sell $\frac{1}{3}$ of my part, what part have I left?

8. $.375 \times 500 \times 2\frac{1}{2} \times 100 \times \frac{1}{3} \times 3000 = ?$

9. Reduce to their lowest terms:

$\frac{144}{230}$ $\frac{144}{288}$ $\frac{154}{196}$ $\frac{31250}{50000}$

Write plainly from memory when a fraction is at its lowest terms.

CXLIII.

1. Write the definition of a mixed number.

Reduce $25\frac{4}{5}$ $37\frac{6}{7}$ $385\frac{5}{8}$

$275\frac{3}{16}$ $484\frac{13}{15}$ $650\frac{9}{12}$

2. $\frac{175}{23}$ $\frac{478}{42}$ $\frac{1575}{125}$ $\frac{2375}{334}$

3. $\frac{10}{15}$ $\frac{20}{45}$ $\frac{36}{48}$ $\frac{64}{72}$ $\frac{33}{77}$

4. How many halves in 5? In 12? In 13? In 25?

5. How many thirds in 6? In 11? In 15?

6. How many 24ths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{3}{4}$? In $\frac{5}{6}$? In $\frac{7}{8}$? In $\frac{5}{12}$?

7. How often is $\frac{2}{3}$ contained in 1? $\frac{3}{4}$ in 1?

8. How often is $\frac{5}{8}$ contained in 1? $\frac{6}{7}$ in 1?

9. How often is $\frac{3}{5}$ contained in 2? $\frac{3}{4}$ in 5?

10. $\frac{1}{3}+3\frac{1}{3}+\frac{5}{3}+4\frac{1}{3}=?$ $\frac{2}{7}+\frac{5}{7}+\frac{6}{7}+5=?$

11. $\frac{5}{8}+7\frac{1}{8}+6\frac{7}{8}+9=?$ $\frac{9}{10}+\frac{5}{10}+5\frac{6}{10}+4=?$

12. Change the following fractions to fractions having 96 for a denominator:

$\frac{7}{12}$ $\frac{5}{8}$ $\frac{5}{32}$ $\frac{7}{16}$ $\frac{27}{48}$

13. Add $\frac{1}{2}$, $\frac{5}{9}$, $\frac{4}{7}$, $\frac{2}{3}$, $\frac{8}{14}$.

CXLIV.

1. Add $2\frac{5}{7}$, $13\frac{2}{3}$, $14\frac{5}{9}$, $25\frac{17}{1}=?$

2. $275\frac{5}{16}-39\frac{4}{7}=?$ $275-39\frac{5}{32}=?$

3. $1873\frac{2}{3}-879=?$ $1873-879\frac{2}{3}=?$

4. $24\frac{3}{4}-18.25=?$ $18.25-16\frac{3}{4}=?$

5. I bought $32\frac{1}{2}$ lbs. of coffee at one time, $44\frac{3}{5}$ lbs. at another, $56\frac{5}{8}$ lbs. at another, and $28\frac{3}{16}$ lbs. at another time. I paid 10 cents a pound for it. I used $87\frac{1}{2}$ pounds and sold the remaining pounds at $13\frac{1}{3}$ cents a pound; did I gain or lose? and how much?

6. $\frac{3}{5}$ of 500=? $\frac{7}{8}$ of 560=? $\frac{7}{12}$ of 960=?

7. 500 is $\frac{5}{6}$ of what number? 960 is $\frac{8}{11}$ of what number?

8. How many inch cubes can be placed on a space 4 by 6 inches?

9. Work the following without putting down a figure: 5 times $2\frac{1}{3}=?$ 6 times $4\frac{1}{5}=?$ 7 times $4\frac{4}{5}=?$ 8 times $5\frac{1}{7}=?$

CXLV.

1. Work the following without putting down a figure: 8 times $\frac{3}{4}$ = ? 12 times $\frac{5}{8}$ = ? 11 times $\frac{8}{9}$ = ?

2. 7 times $\frac{4}{7}$ = ? 10 times $\frac{5}{6}$ = ? 6 times $\frac{6}{7}$ = ?

3. $\frac{3}{8} \times \frac{4}{6} \times \frac{7}{9}$ = ? $2\frac{1}{2} \times \frac{5}{6} \times \frac{3}{25}$ = ? $\frac{4}{9} \times \frac{27}{32} \times \frac{1}{7}$ = ?

4. Write the two rules for the division of common fractions. Work the following both ways:

$\frac{3}{8} \div \frac{1}{2}$ = ? $\frac{4}{7} \div 2$ = ? $2 \div \frac{4}{7}$ = ?

5. $3\frac{1}{3} \div \frac{2}{3}$ = ? $4\frac{1}{8} \div 3$ = ? $\frac{1}{3} \div 3\frac{1}{8}$ = ?

6. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{6}{7}$ divided by $\frac{3}{7}$ of $1\frac{1}{4}$ of $2\frac{1}{5}$.

7. $\frac{5\frac{1}{2}}{6\frac{3}{5}}$ $\qquad$ $\frac{8\frac{1}{2}}{17}$ $\qquad$ $\frac{6\frac{1}{4}}{12\frac{1}{2}}$

8. How many dozen eggs at $12\frac{1}{2}$ cents a dozen will pay for the following groceries:

8	lbs. coffee	at $33\frac{1}{3}$ cents a pound.
15	lbs. sugar	at $6\frac{1}{4}$ cents a pound.
$13\frac{1}{2}$	lbs. bacon	at $11\frac{1}{2}$ cents a pound.
24	cans baking powder	at 45 cents a can.

CXLVI.

1. Add 7 tenths; 33 hundredths; 20 units and 20 thousandths; 1875 thousandths; 76 units and 76 millionths; 500 units and 1875 hundred thousandths.

2. These are the fractions whose decimals you must carry in your memory: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{3}{16}$. Try to call the decimal for each fraction without working them.

3. Reduce $\frac{1}{3}$, $\frac{4}{7}$, $\frac{5}{9}$ to decimals having four decimal places, and find their sum. *Ans.* $1.4603\frac{11}{63}$.

4. Reduce the following to common fractions and to their lowest terms:

.8	.84	.064	.035
.025	.1875	.3125	.04375

5. Reduce the following to decimals:

$\frac{3}{5}$ $\frac{13}{16}$ $\frac{16}{25}$ $\frac{7}{12}$ $\frac{19}{32}$

CXLVII.

1. Find the prime factors in the following numbers: 242, 1080, 1225, 1540, 1040.

2. What is a factor? What is a prime factor?

3. Find the G. C. D. of 630 and 490. Of 576 and 480. Of 312, 144, 120.

4. What is the least common multiple of several numbers?

Do you remember the following principles? *The G. C. D. of several numbers cannot be larger than the smallest number. The L. C. M. of several numbers cannot be smaller than the largest nnmber.*

5. What effect does placing a cipher on the right of these decimals have? .9 .15 .125

Explain each one.

6. What effect does erasing the ciphers on the right of these decimals have? .20 .350 .470 .200

CXLVIII.

1. Multiply 37.006 by 250. Prove.

2. Multiply 9.0026 by 3.4 Prove.

3. Multiply .0375 by 1.25 Prove.

4. If I pay $7.84 for a barrel of flour, what is the value of 1 pound?

5. If I pay $15 for a barrel of pork, what is the cost of 1 pound?

6. What part of one pound is 12 ounces?

7. What part of 1 hogshead is 27 gallons?

8. What is the cost of 75 lbs., 8 oz. of coffee at 35 cents a pound? *Ans.* $26.425.

9. What is the cost of 15 hhds., 35 gal. of molasses at $45 a hogshead? *Ans.* $710.

CXLIX.

1. I mixed 20 lbs. of sugar at 12 cents per lb., 30 lbs. at 15 cents per lb., 15 lbs. at 10 cents per pound, 24 lbs. at 20 cents per lb.; what was a pound of the mixture worth?

2. A man had $2500, he gave his oldest son $\frac{1}{5}$, his youngest son $\frac{1}{5}$ of the remainder, his daughter $\frac{1}{4}$ of the remainder, the balance to his wife; how much did each receive?

3. I bought an equal number of pounds of tea, coffee, and salt for $99; the tea was 60 cents a pound, the coffee 35 cents, and salt 4 cents: how many pounds of each did I buy? Prove your work.

CL.

1. In an orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{8}$ pears, $\frac{1}{9}$ plums, $\frac{1}{12}$ cherries, and 26 trees bear peaches; how many trees in the orchard? and how many trees of each kind?

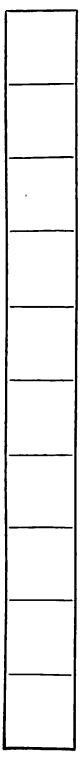
2. How many cords of wood at $3 a cord will pay for $4\frac{1}{2}$ barrels of cider at $3\frac{4}{5}$ a barrel?

3. If $\frac{1}{3}$ of a pint of vinegar is worth $\frac{1}{4}$ of a dime, how many pints of vinegar can you buy with $1.50?

4. A man paid $72 for a writing desk, $\frac{3}{8}$ of the price of the desk was $\frac{9}{10}$ of what he paid for a bureau, and $\frac{5}{8}$ of what he paid for the bureau was $\frac{5}{8}$ of what he paid for a dining table; what was the cost of the table?

THE METRIC SYSTEM.

The Metric System is a sytem of measures of lengths, surfaces, volumes, capacities, and weights, invented by the French, and ordered by law into compulsory use in France in 1841. Since then it has been adopted by other countries, and by an act of the American Congress in 1866 was legalized in the United States. As yet, it is used only in our custom houses and by men of science.



The METER—39.37 inches in length—is the basis of the system, from which all the units of the measures are derived. Thus:

The *meter* is used for lengths and distances.

The *square meter* is used for surfaces.

The *cubic meter* for large volumes.

The *liter*—a cubic decimeter—for small volumes.

The *gram*—a cubic centimeter—for weights.

On the margin we give a decimeter, from which the teacher may construct a meter. Just as the unit—one dollar—of our currency is increased and decreased by multiplying or dividing by 10, so in the metric measures the denominations of a measure are produced by multiplying or dividing the unit of measure by 10 or 100 or 1000, &c.

The subdivisions of the unit of measure are indicated by certain Latin prefixes; thus, *deci* $\frac{1}{10}$ = .1; *centi* $\frac{1}{100}$ = .01; *mille* $\frac{1}{1000}$ = .001.

The value of the multiples of the unit of measure is indicated by certain Greek prefixes; thus, *deka*, 10; *hekto*, 100; *kilo*, 1000; *myria*, 10000.

TABLE OF LENGTHS.

Latin prefixes indicate subdivisions of the unit.

1 millimeter mm=$\frac{1}{1000}$=.001 of a meter.
1 centimeter cm=$\frac{1}{100}$= .01 of a meter.
1 decimeter dm= $\frac{1}{10}$ = .1 of a meter.

1 METER—The unit=39.37 inches.

Greek prefixes indicate multiples of the unit.

1 dekameter Dm = 10 meters.
1 hektometer Hm = 100 meters.
1 kilometer Km = 1000 meters.
1 myriameter Mm =10000 meters.

(The pupil should notice in this and other tables the difference between *deci* $\frac{1}{10}$, and *deka* 10, the latter being 100 times greater than the former.)

It will be observed that multiples and subdivisions of the unit of measure are separated by a decimal point just as we separate dollars and cents. To change 250 decimeters to meters, it may be divided by 10, but we may more conveniently place a decimal point between "5" and "0"; thus 250 dm=25.0 m.

To change hektometers to meters we may multiply by 100, or, more conveniently, annex two ciphers; 25 hektometers=2500 meters.

The pupil will learn this rule :

To change a number to a lower denomination move the point to the right.

To change a number to a higher denomination move the point to the left.

28.08 m = 28 meters and 8 centimeters = 2.808 dekameters = 280.8 decimeters = .2808 hektometers = 2808 centimeters.

Again with 850.5 meters.

850.5 meters = 8.505 hektometers = 85.05 dekameters = .8505 kilometers = 8505 decimeters = 85050 centimeters = 850500 millimeters.

The teacher should repeat this exercise until the pupil can work the table with facility.

1. Change 4.375 m to centimeters. Ans. 437.5 cm.

2. Change 5052.5 m to kilometers. Ans. 5.0525 Km.

3. 4.8 Km to meters. Ans. 4800 m.

4. 80 Mm to meters. Ans. .080 m.

The pupil should be required to construct a table of square measure from the preceding TABLE of LENGTH, remembering that the denominations increase from square millimeters by multiples of 100, not of 10.

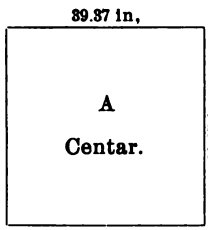
(The square meter and its subdivisions are used to measure small surfaces.)

TABLE OF LAND MEASURE.

The square meter is called a centar.

100 centars (ca) make 1 ar (a).
100 ars make 1 hektar (Ha).

The ar is the unit of measure.



An ar contains a little over 119½ square yards.

A hektar contains 100 × 119½ square yards, or about 2½ acres.

1. What part of a square meter is a square decimeter? How many square meters in a square kilometer?

2. How many acres in 10 hectars?

3. How many acres in 1000 ars? Ans. 24.7 acres.

Table of Measure of Capacity.

The Unit of Capacity is the Liter (lēter.)

A liter is equal to a cubic volume of water one decimeter or $\frac{1}{10}$ of a meter or nearly four inches on each edge. The liter and hektoliter are most commonly used—the liter, which is about 1 qt., for measuring wine, milk, &c.; the hektoliter, about a half peck less than 3 bushels, for measuring grain, fruit, roots, &c.

10 centiliters (cl) make 1 deciliter (dl).
10 deciliters make 1 Liter—The Unit.
10 liters make 1 dekaliter (Dl).
10 dekaliters make 1 hektoliter (Hl).

For measuring wood a cubic meter is used called the *ster*, about 1⅓ cubic yards.

1. Change 445.3 l to hektoliters. Ans. 4.456 Hl.
2. Change 2800 l to cubic meters. Ans. 2.4 m^3.
3. How many l in 1 hektoliter?
4. How many l in 1 cubic meter?

The pupil should be required to construct a Table of Cubic Measure from the Table of Length, remembering that the denominations increase from the cubic millimeter by multiples of 1000.

To find solid contents reduce the measure of the dimensions to meters, and multiply the product of the dimensions, which will be cubic meters, by 1⅓ yards.

TABLE OF MEASURES OF WEIGHT.

The unit of Weight is the *Gram.* It has the weight of a cubic volume of water equal to one cubic centimeter, or about 15½ grains Troy. The kilogram is 1,000 grams. The metric ton, 1,000 kilograms. These are the weights most commonly used. The gram is used for very exact weight, as in weighing medicine, jewels, &c. The kilogram is used for weighing groceries, &c., and the ton for hay and other very heavy articles. The kilogram is equal to 2⅕ pounds. The ton equals 2,200 pounds avoirdupois.

10 milligrams (mg) make . . . 1 centigram (cg).
10 centigrams make . . . . . 1 decigram (dg).
10 decigrams make . . . . . 1 *gram* (g).
10 grams make. . . . . 1 dekagram (Dg).
10 dekagrams make . . . . . 1 hektogram (Hg).
10 hektograms make . . . . . 1 kilogram (Kg).
10 kilograms make. . . . . 1 myriagram (Mg).
10 myriagrams make. . . . . 1 quintal (Q).
10 quintals or 1000 kilograms . . 1 metric ton (M T).

Problems in this table are worked as in our decimal system.

We present the approximate values of the most commonly used measures of the metric system. It is believed that this information, with the preceding exposition of the system, will enable any thoughtful pupil to go from the French system to our system of compound numbers without great difficulty.

Meter equals	3 feet, 3⅜ inches.
Square meter	10¾ square feet.
Kilometer	⅝ of a mile.
Ar	4 square rods.
Hektar	2½ acres.
Cubic meter	35¼ cubic feet.
3 cubic meters	4 cubic yards.
Ster	¼ of a cord.
Liter	1 quart.
Hektoliter	2⅞ bushels.
Gram	15½ grains.
Kilogram	2⅕ pounds.
Ton	2,200 pounds.